

Top

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las lecciones en pdf están en:



<https://www.physics.umd.edu/rgroups/amo/orozco/results/2023/Results23.htm>

Correlaciones en óptica y en óptica cuántica:

Una serie de lecciones a cerca de
correlaciones y coherencia Junio 2023

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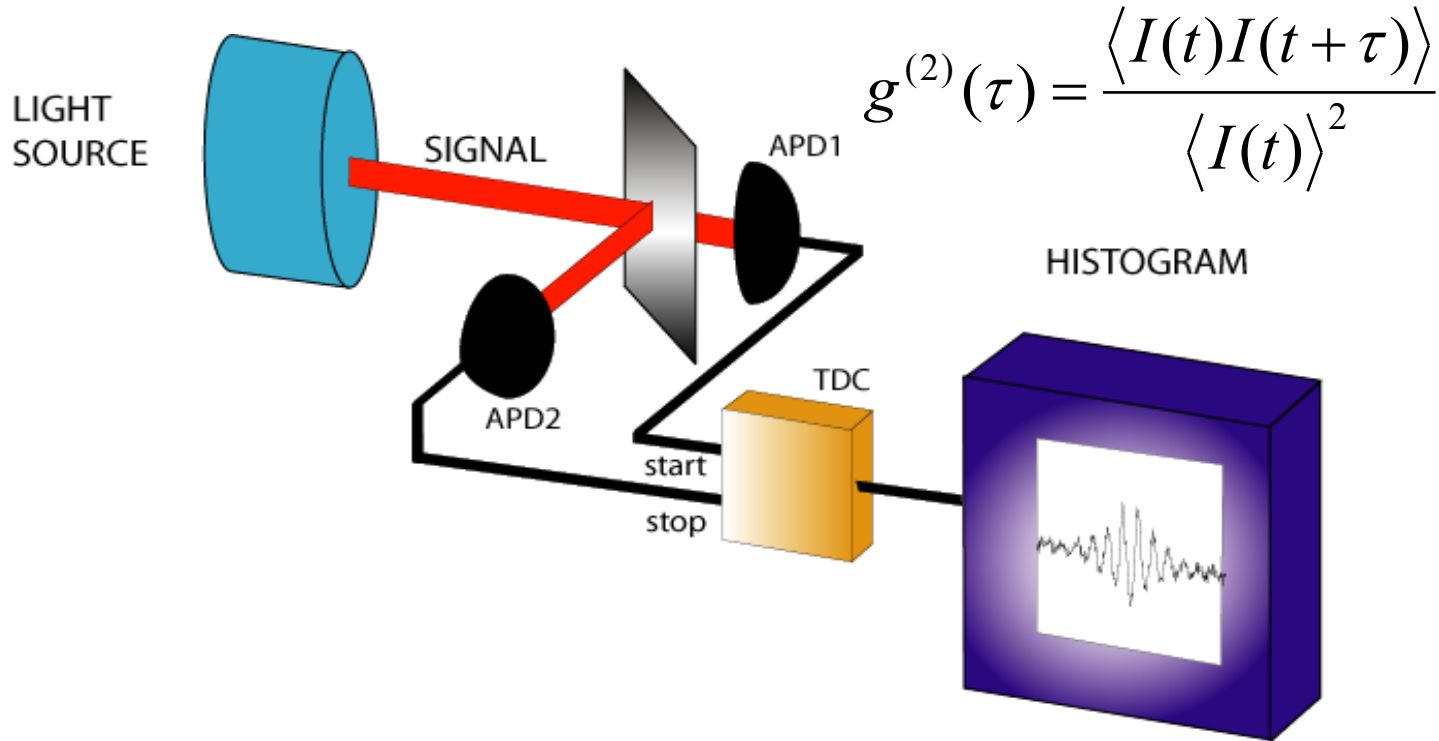
Lección 8

Lista tentativa de tópicos a tratar:

- De la estadística y el algebra lineal a la densidad espectral de potencia.
- Perspectiva histórica y ejemplos en varias areas de física.
- Funciones de correlación en óptica clásica
- La relación entre correlación y la coherencia.
- Funciones de correlación en óptica cuántica.
- Electrodinámica cuántica de cavidades.
- Correlaciones en electrodinámica cuántica de cavidades I: $g^{(3/2)}$.
- Correlaciones en electrodinámica cuántica de cavidades II $g^{(2)}$.

Lección 8

Hanbury Brown and Twiss; Correlación de intensidad-intensidad



R. Hanbury Brown and R.Q. Twiss, Correlation between Photons in Two Coherent Beams of Light, Nature 177, 27 (1956).

Correlaciones cuánticas (Glauber):

$$g^{(2)}(\tau) = \frac{\langle : \hat{I}(t) \hat{I}(t + \tau) : \rangle}{\langle \hat{I}(t) \rangle^2}$$

$$g^{(2)}(\tau) = \frac{\langle : \hat{I}(\tau) : \rangle_c}{\langle : \hat{I} : \rangle}$$

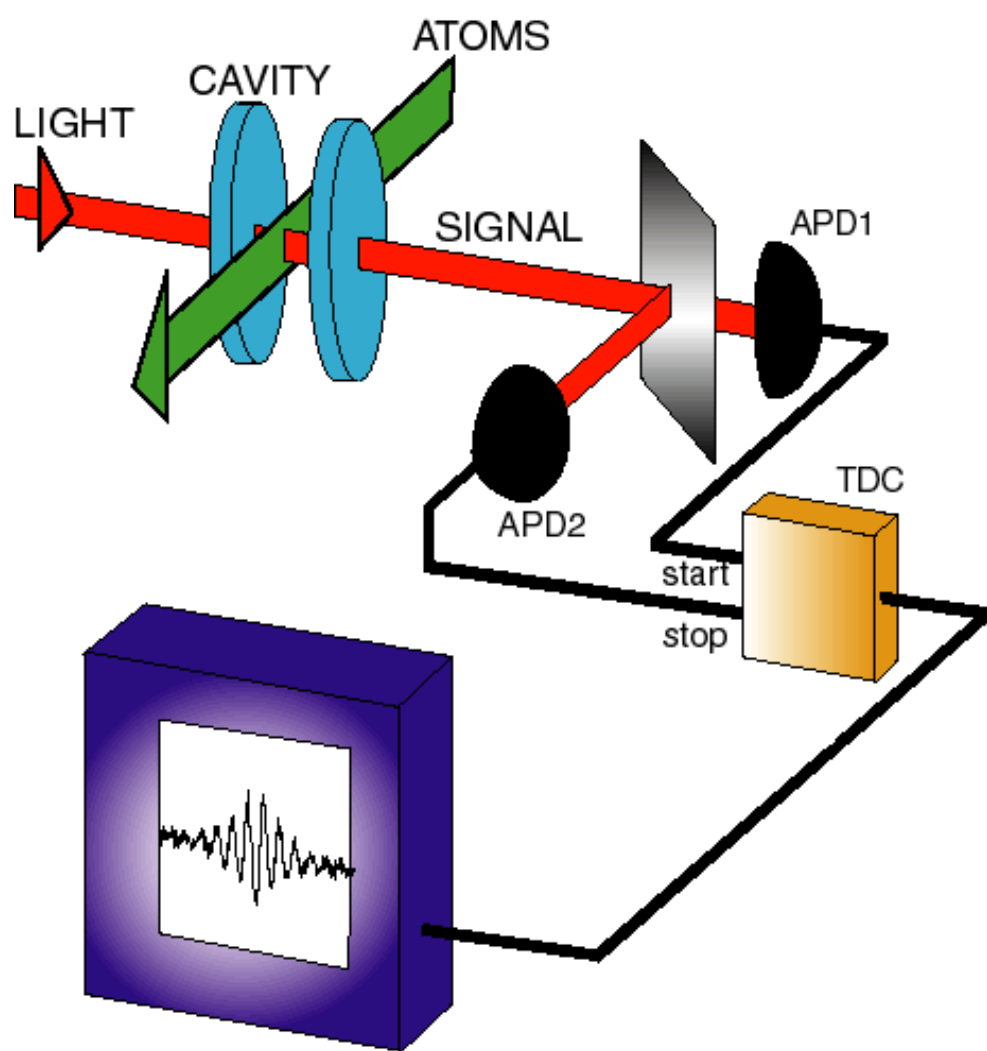
Si detectamos un fotón a tiempo t , $g^{(2)}(\tau)$ da la probabilidad de detectar un segundo fotón después de un tiempo τ .

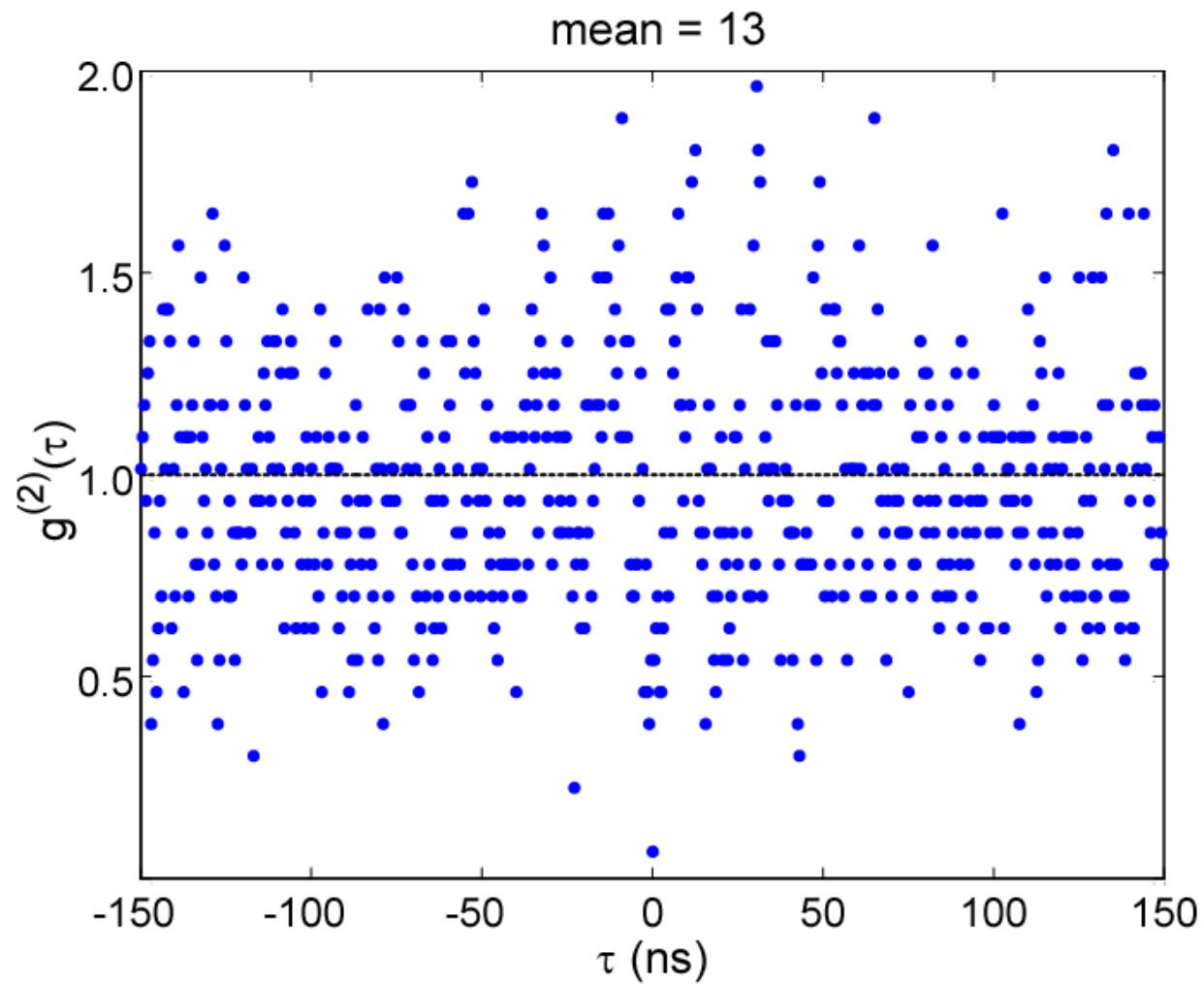
La correlación funciona como mediciones
condicionales en óptica cuántica.

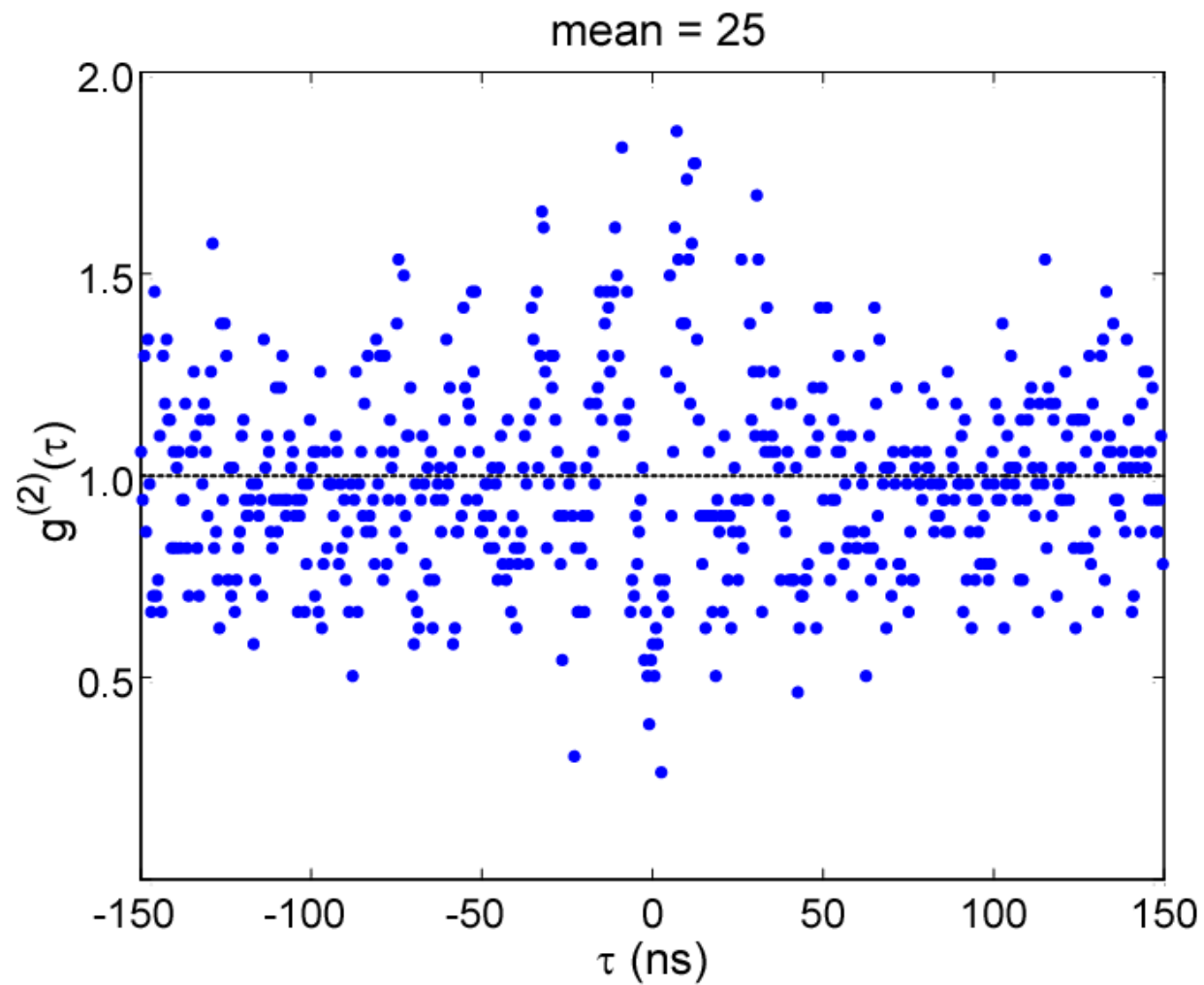
La detección del primer fotón da el estado inicial
que va a evolucionar en el tiempo.

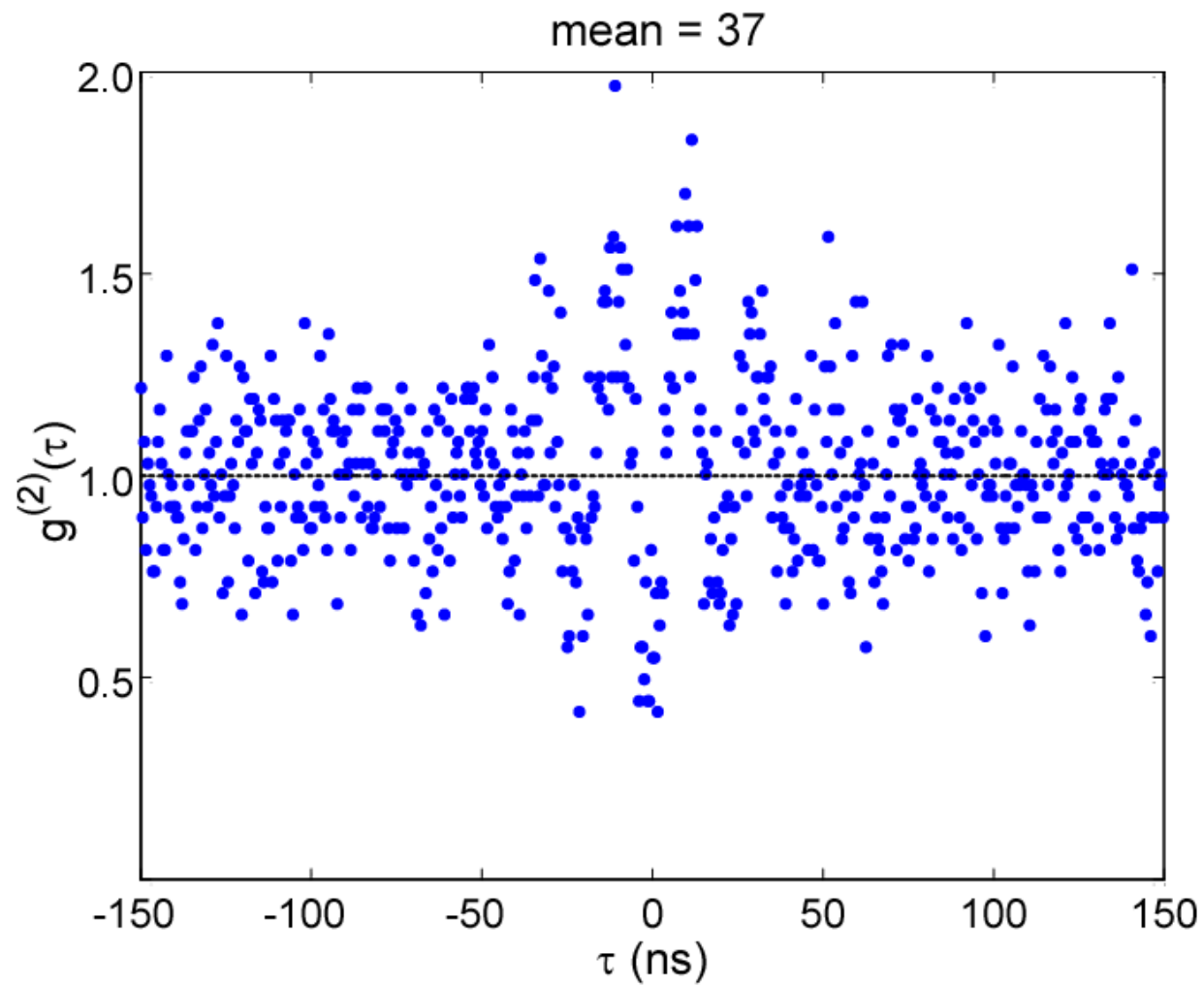
Esto puede sonar como probabilidades
bayesianas.

$g^{(2)}(\tau)$ Hanbury-Brown and Twiss.

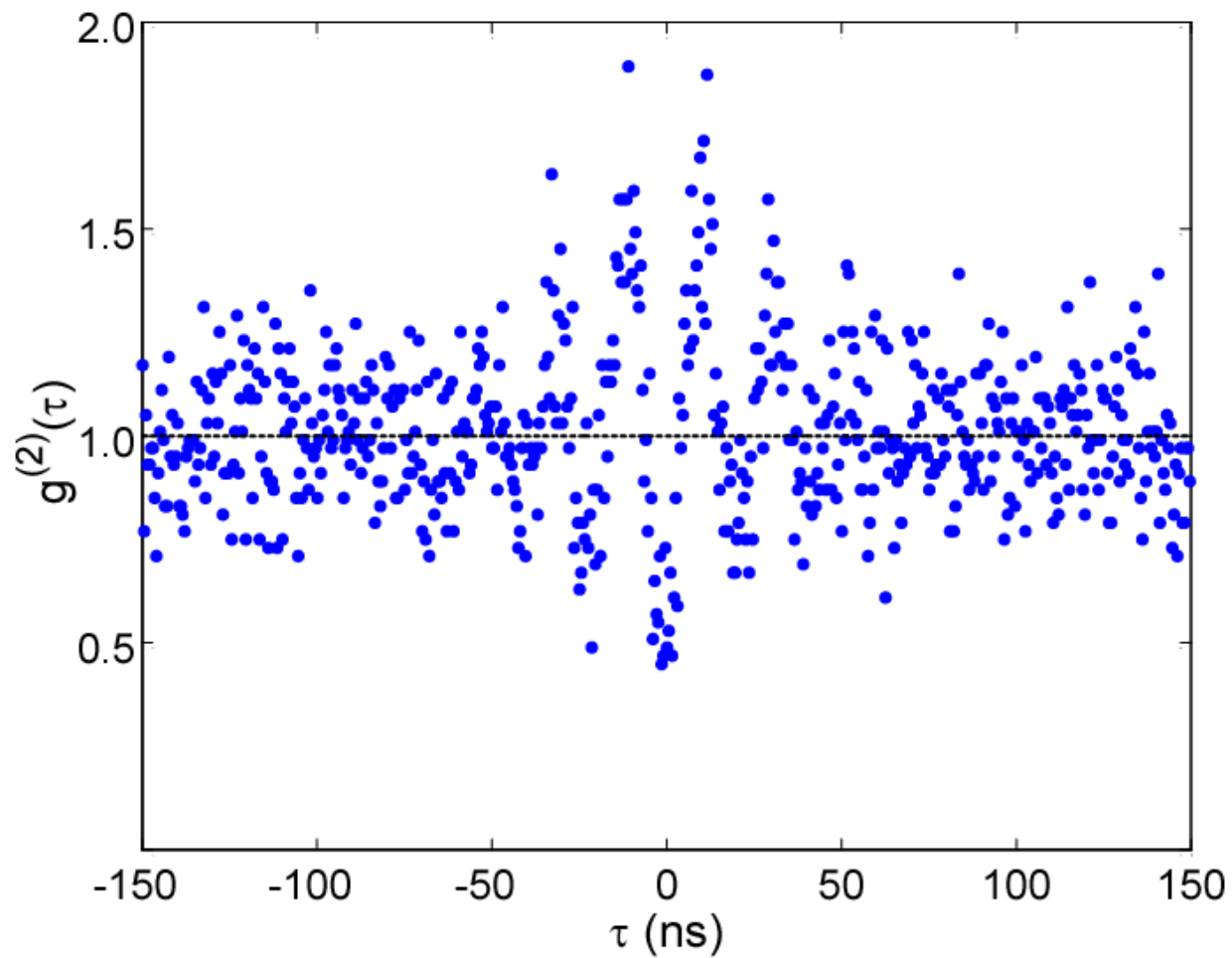


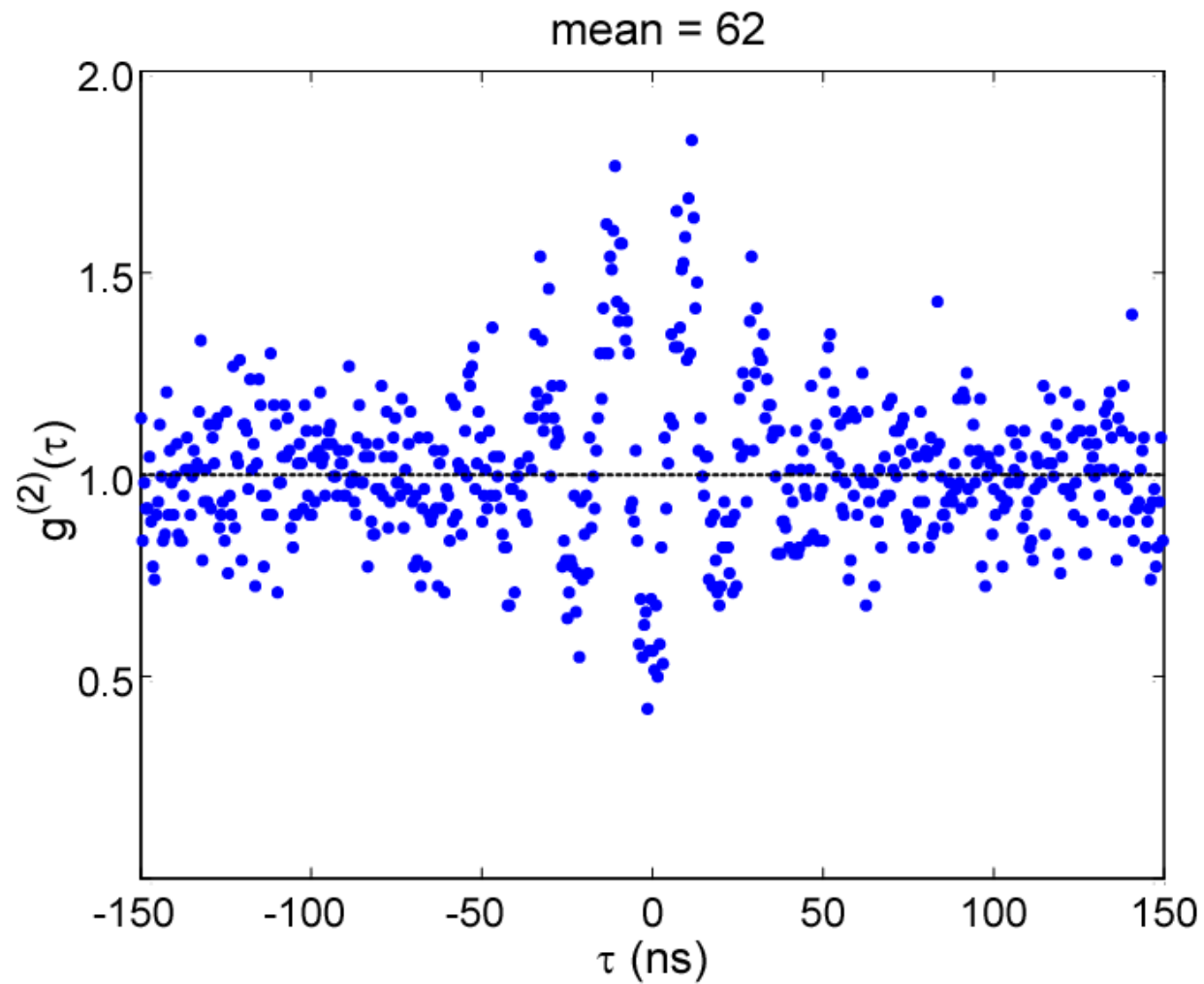




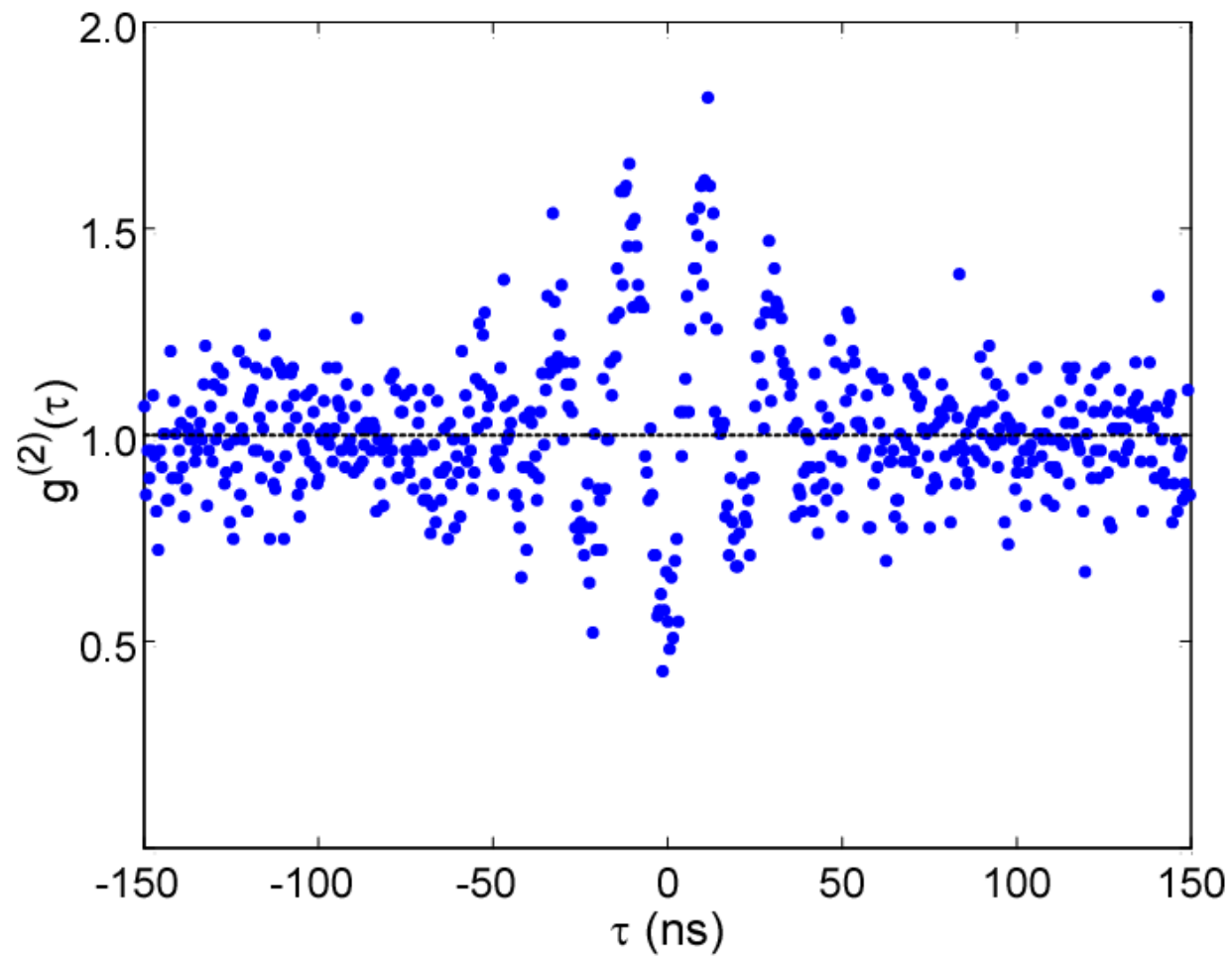


mean = 50

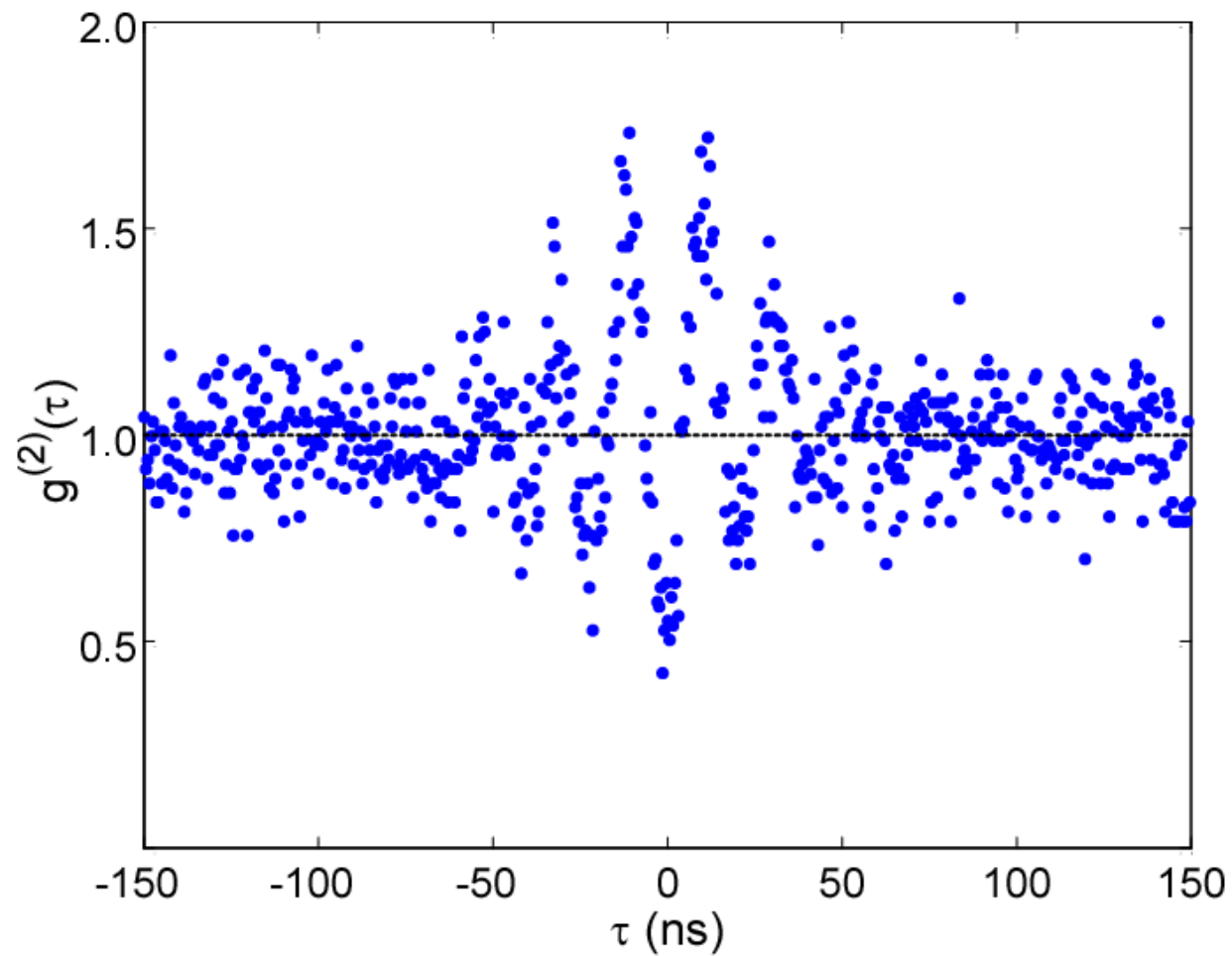




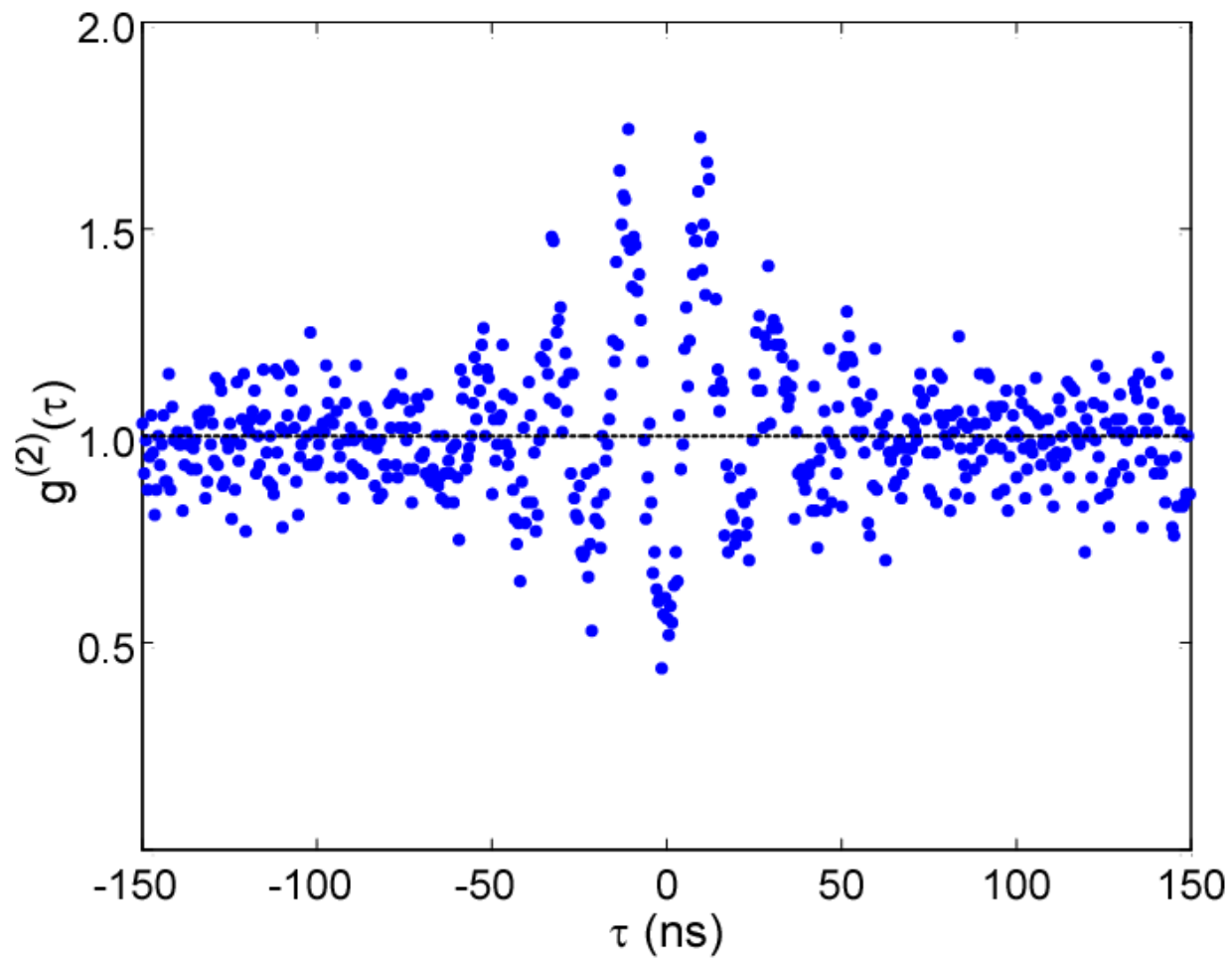
mean = 75



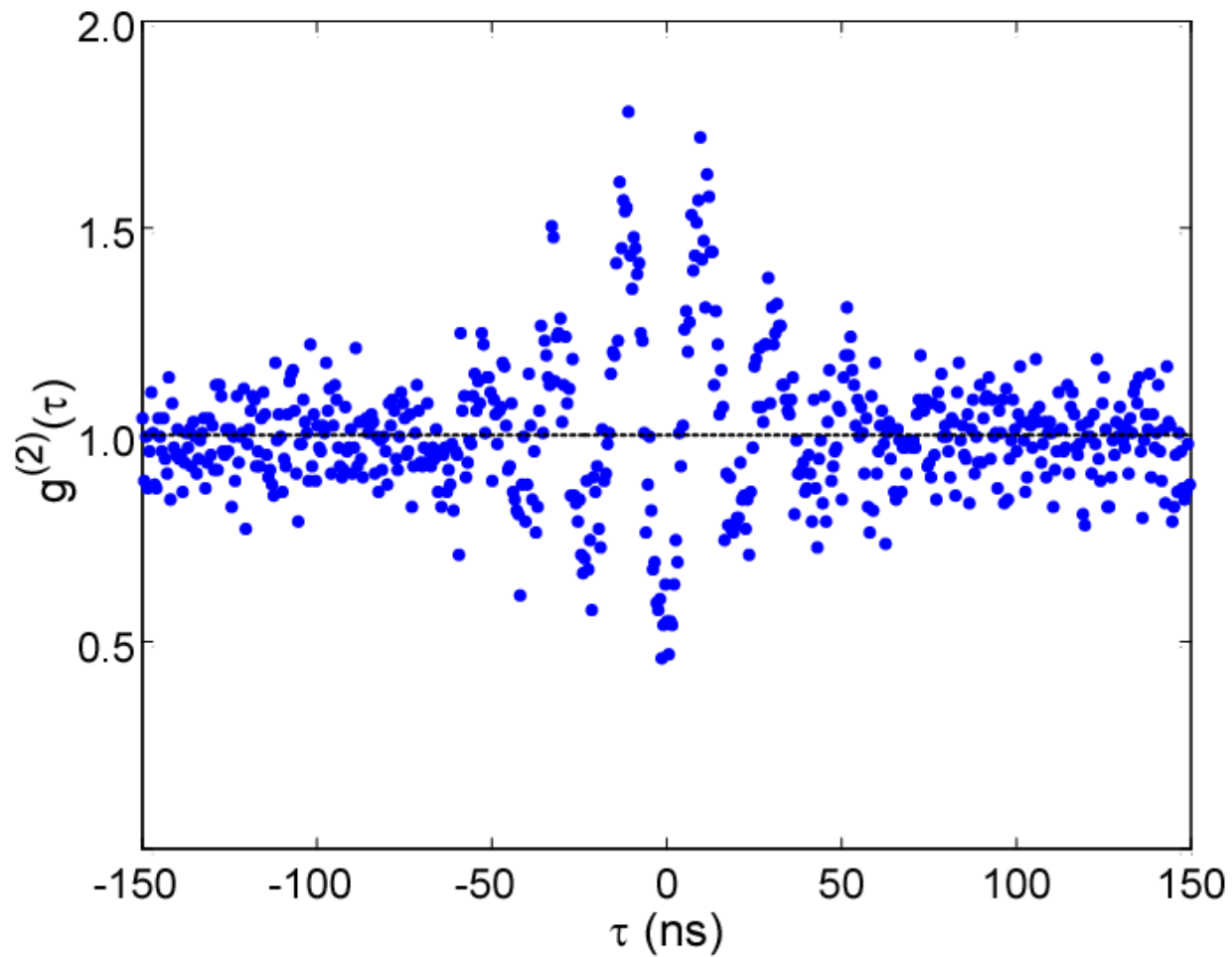
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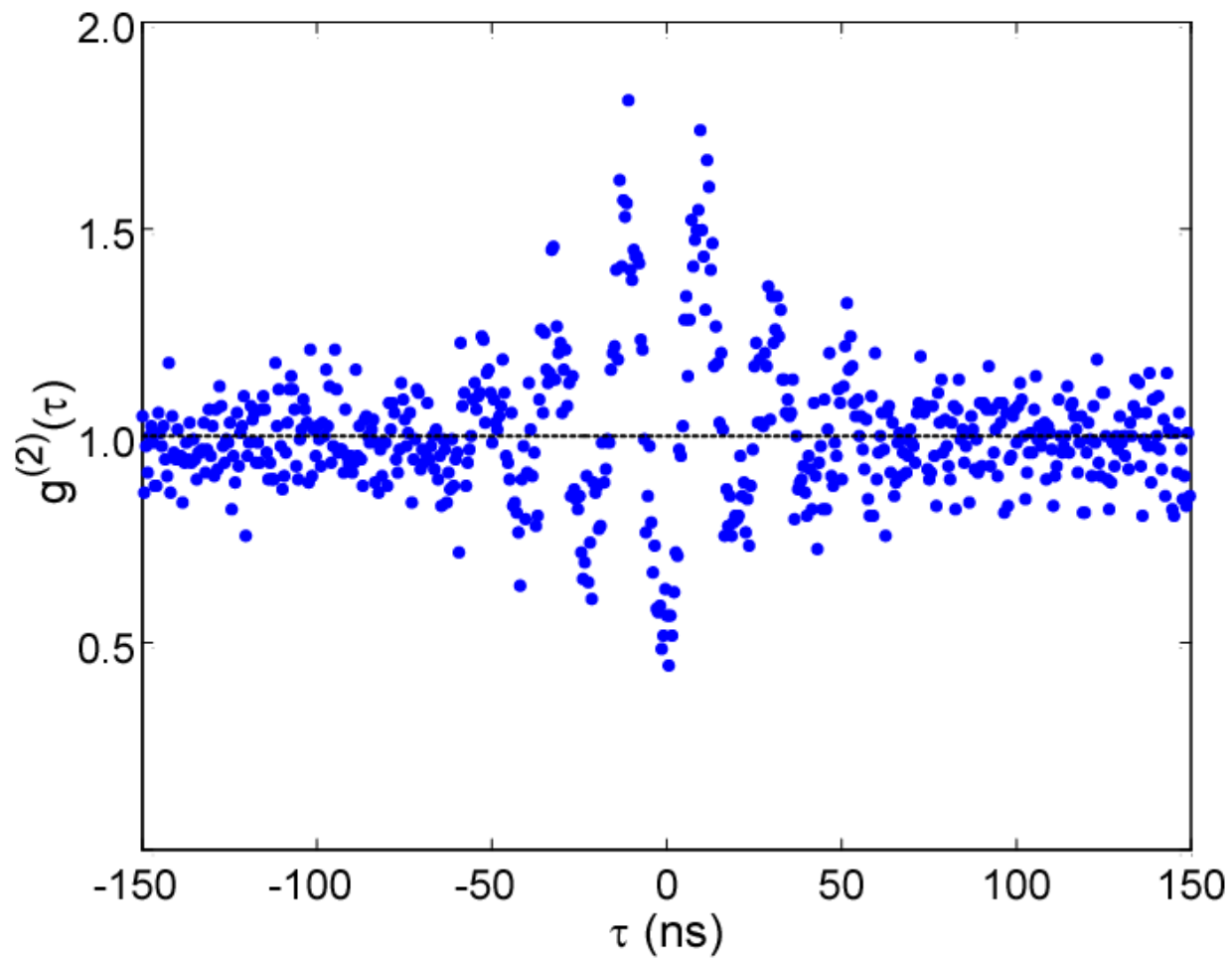
mean = 100



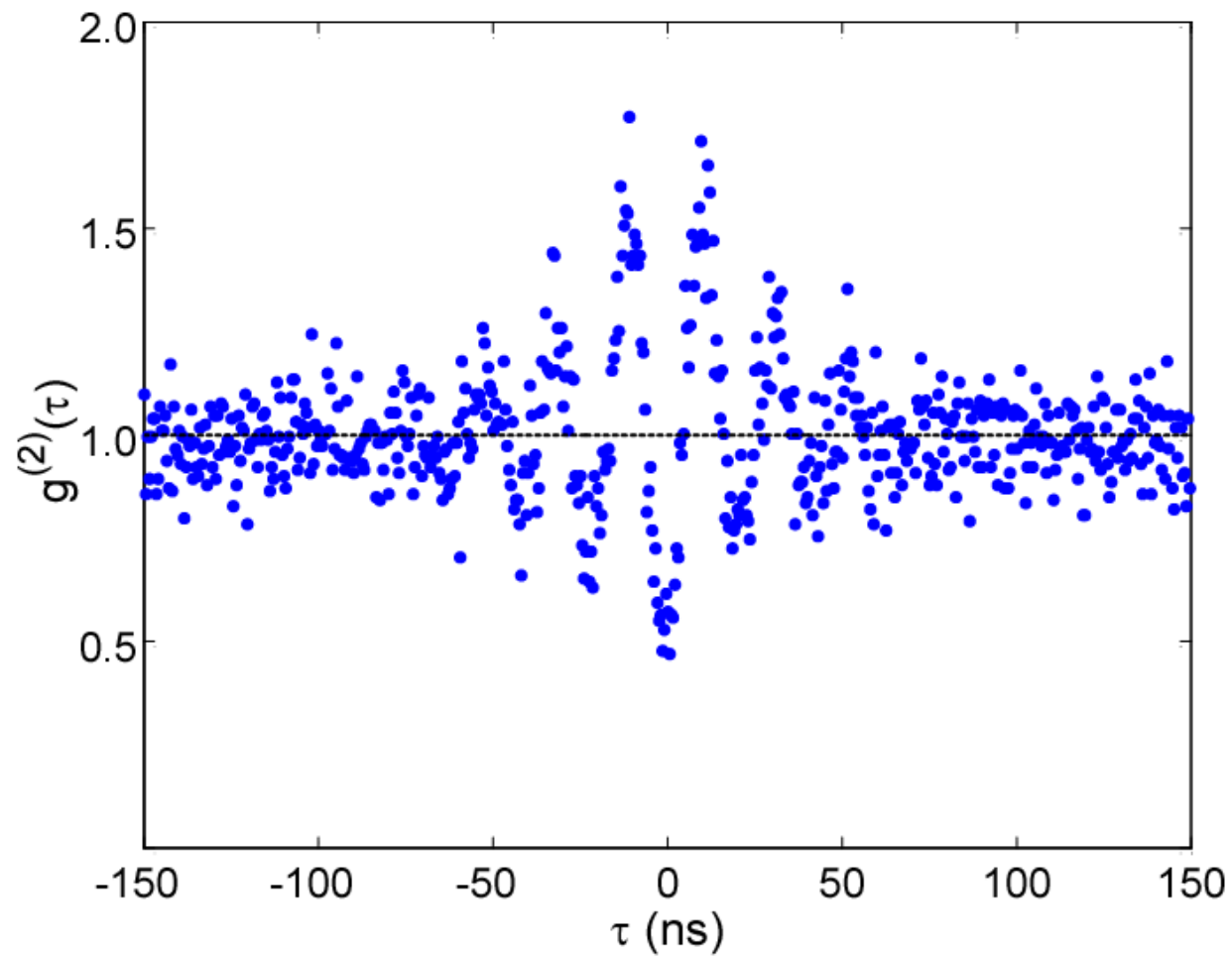
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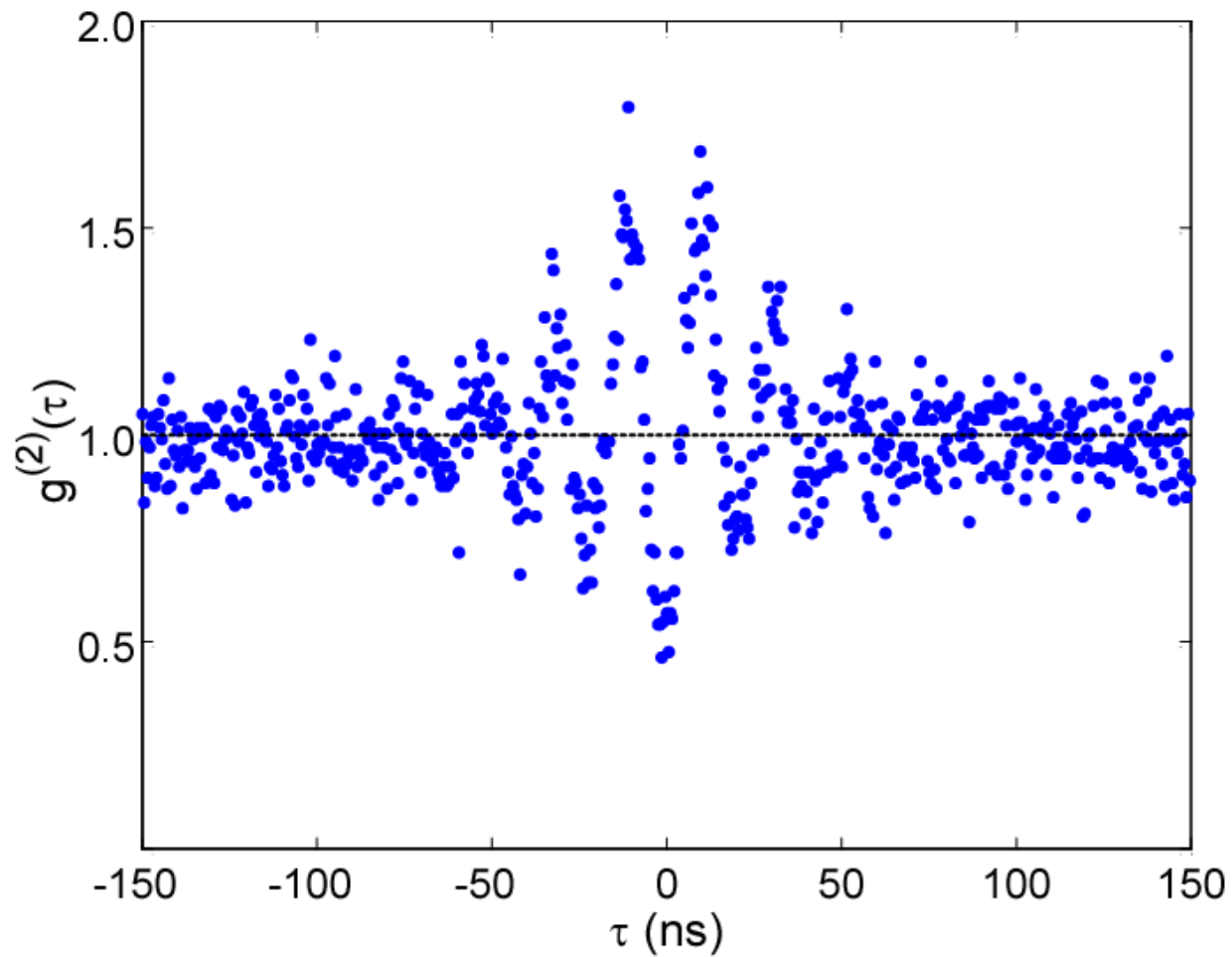
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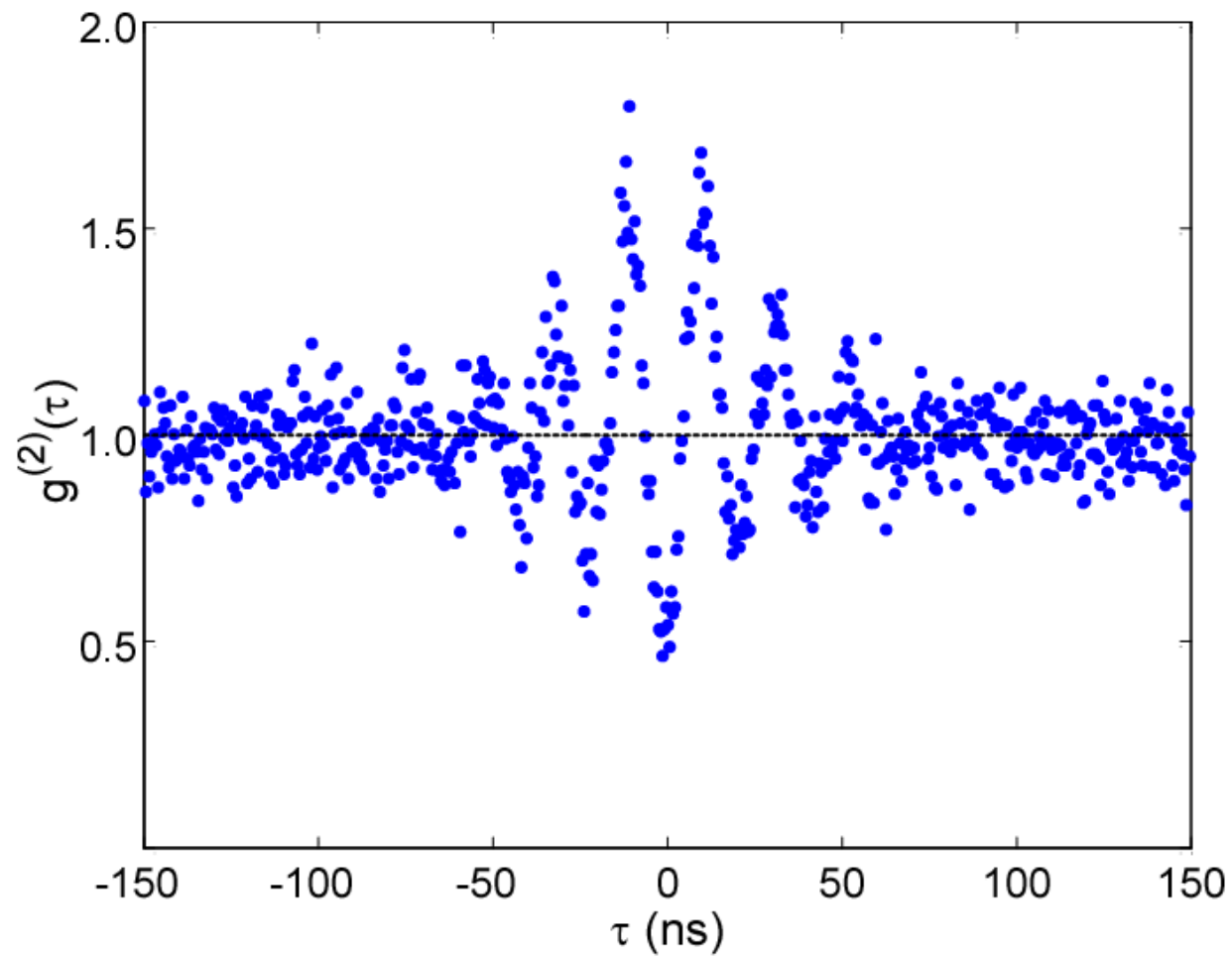
mean = 137



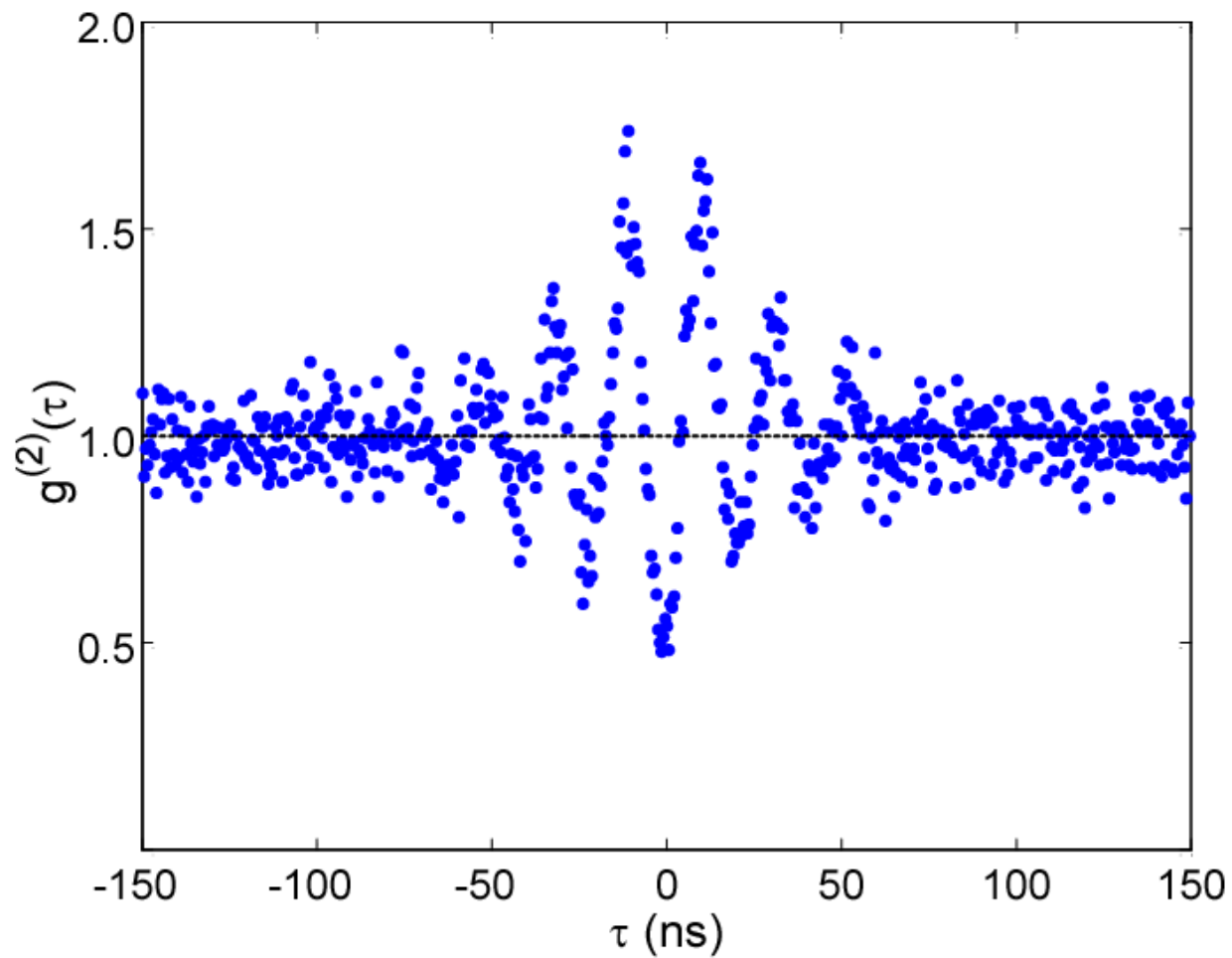
mean = 150



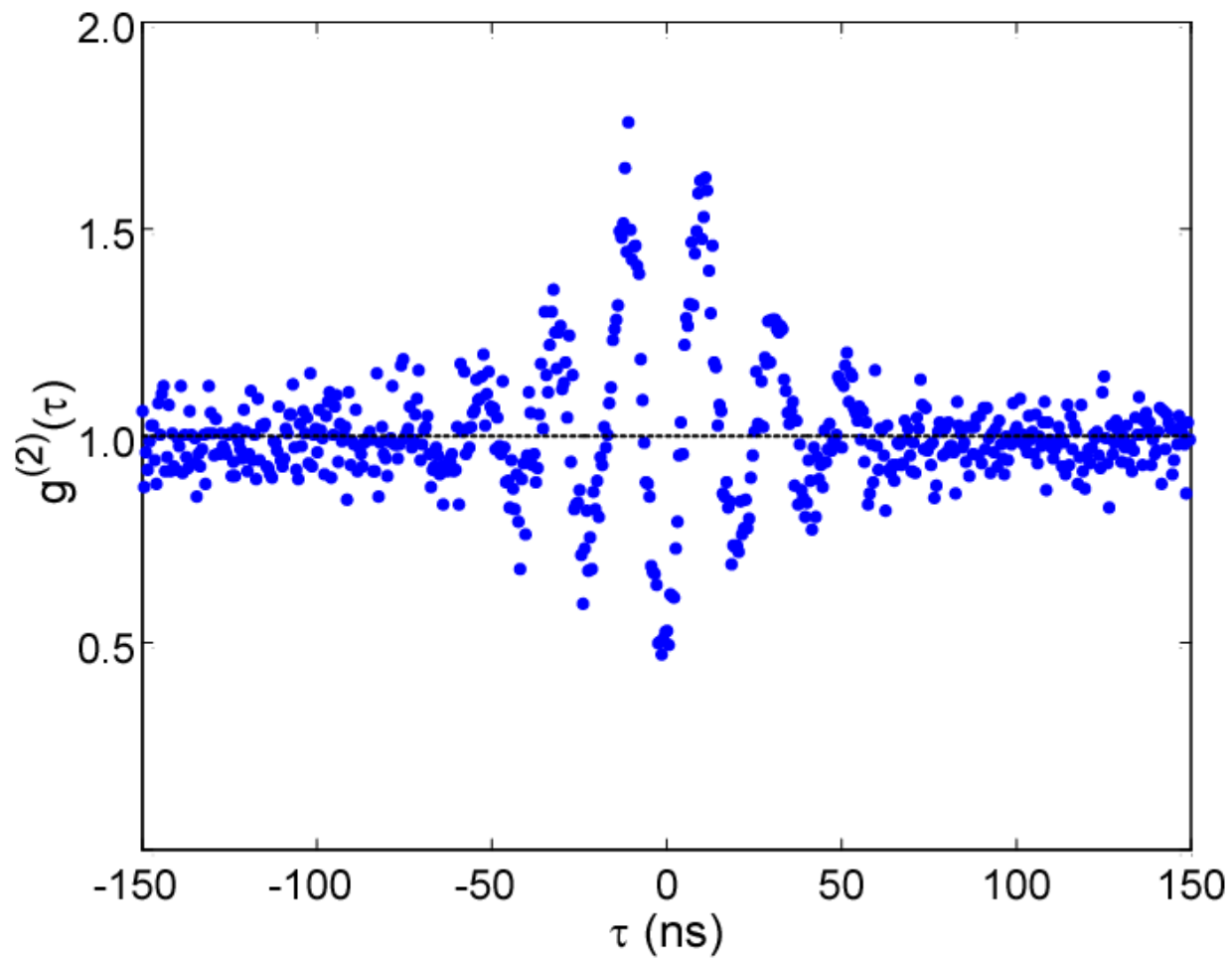
mean = 186



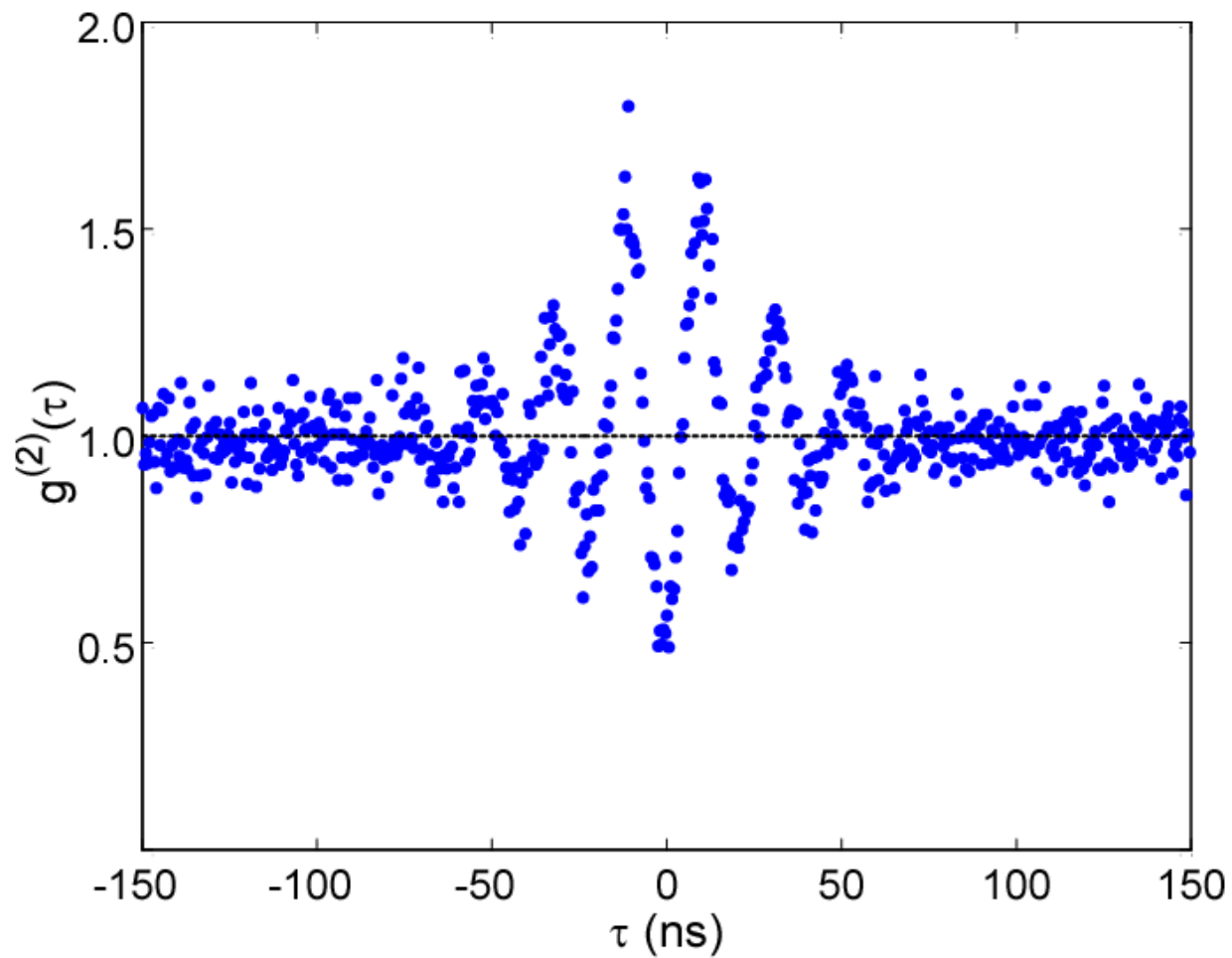
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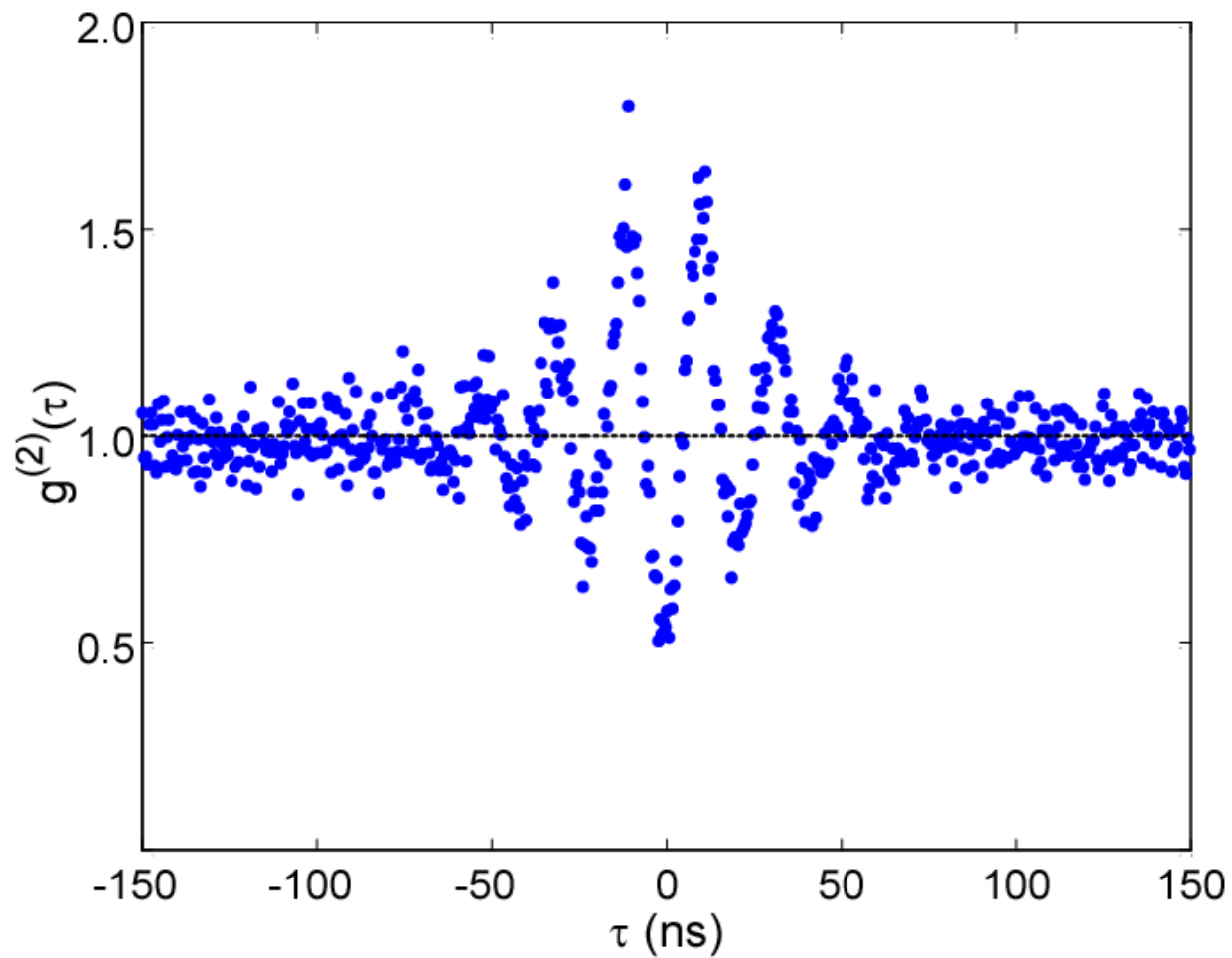
mean = 262



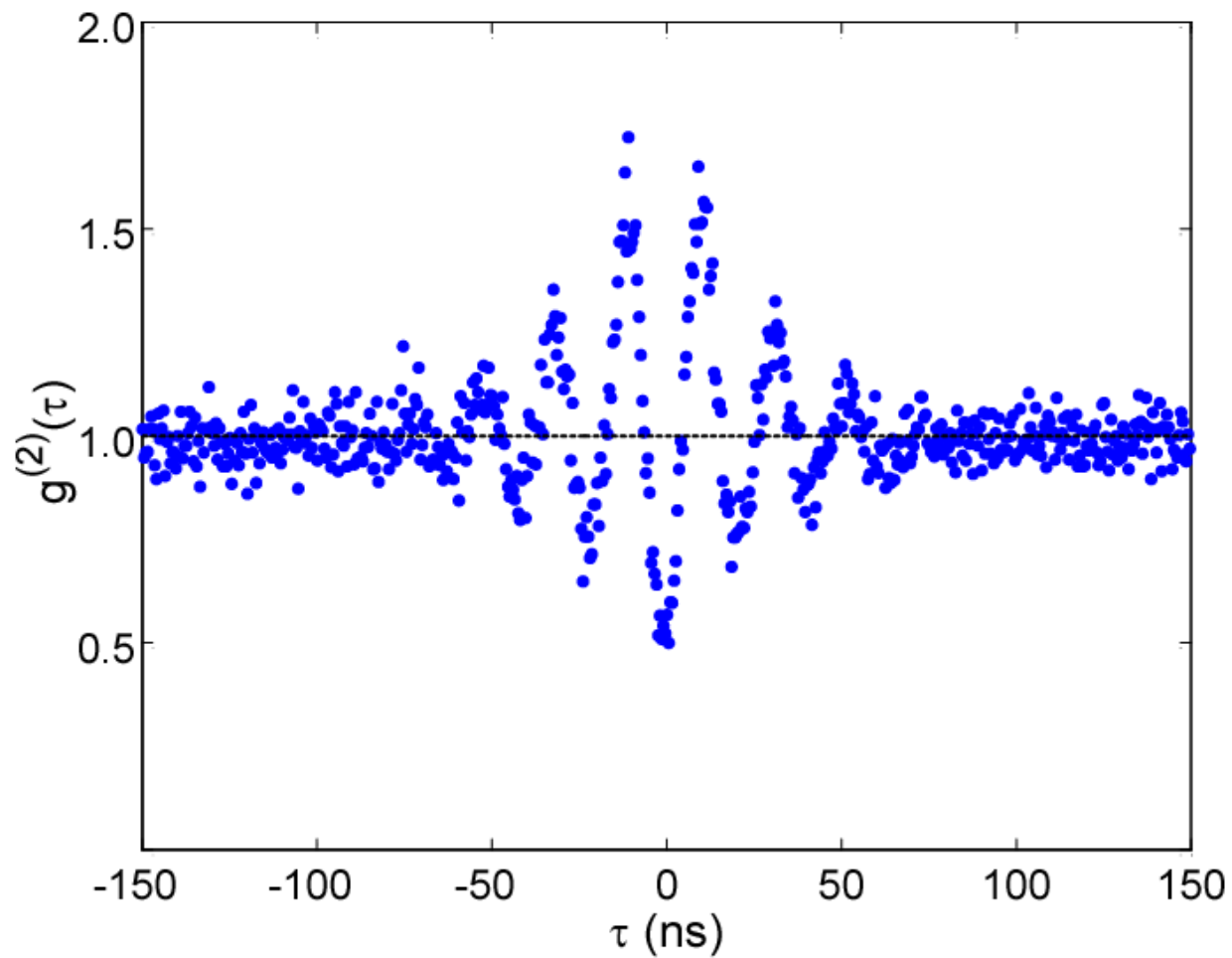
mean = 299



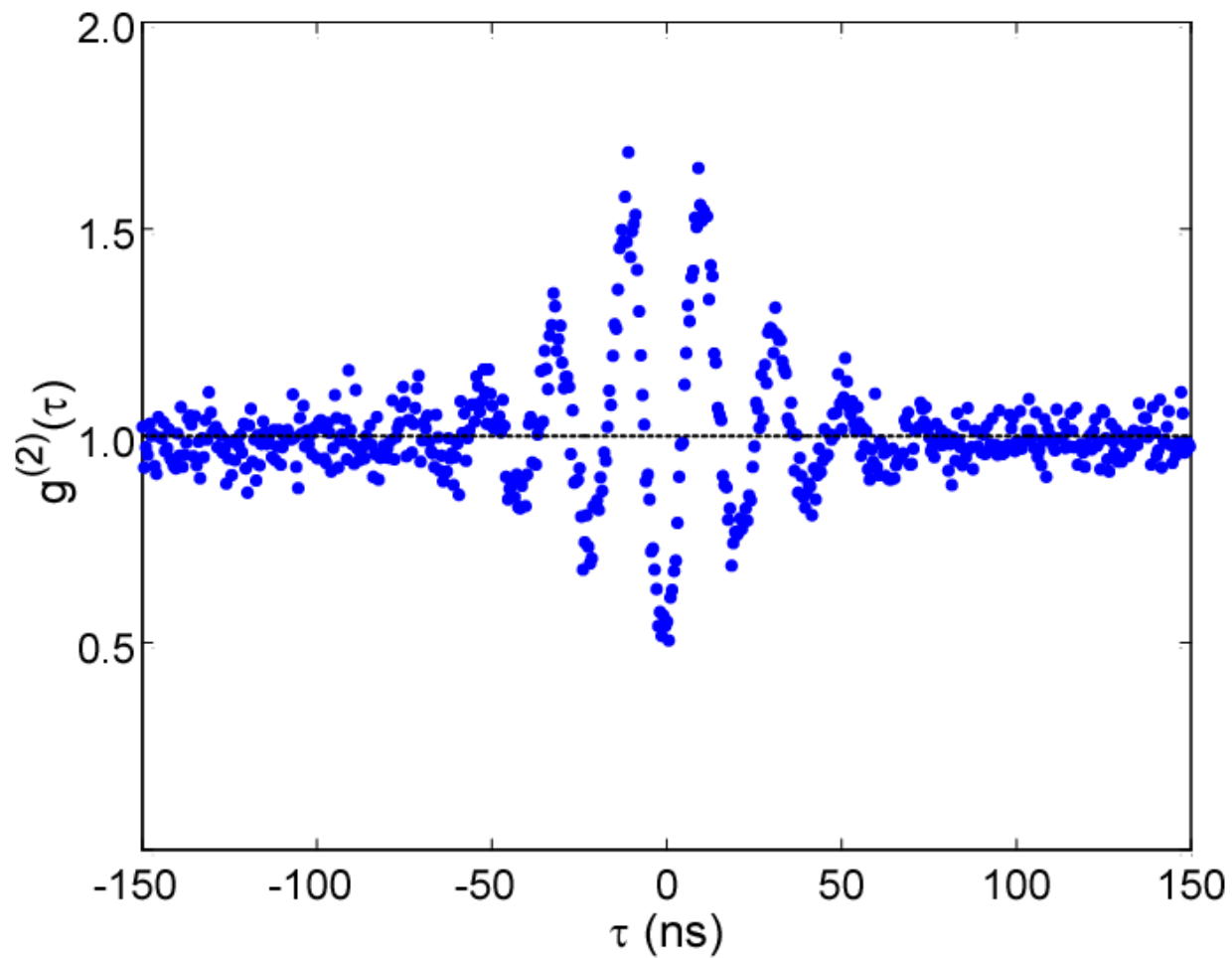
mean = 362



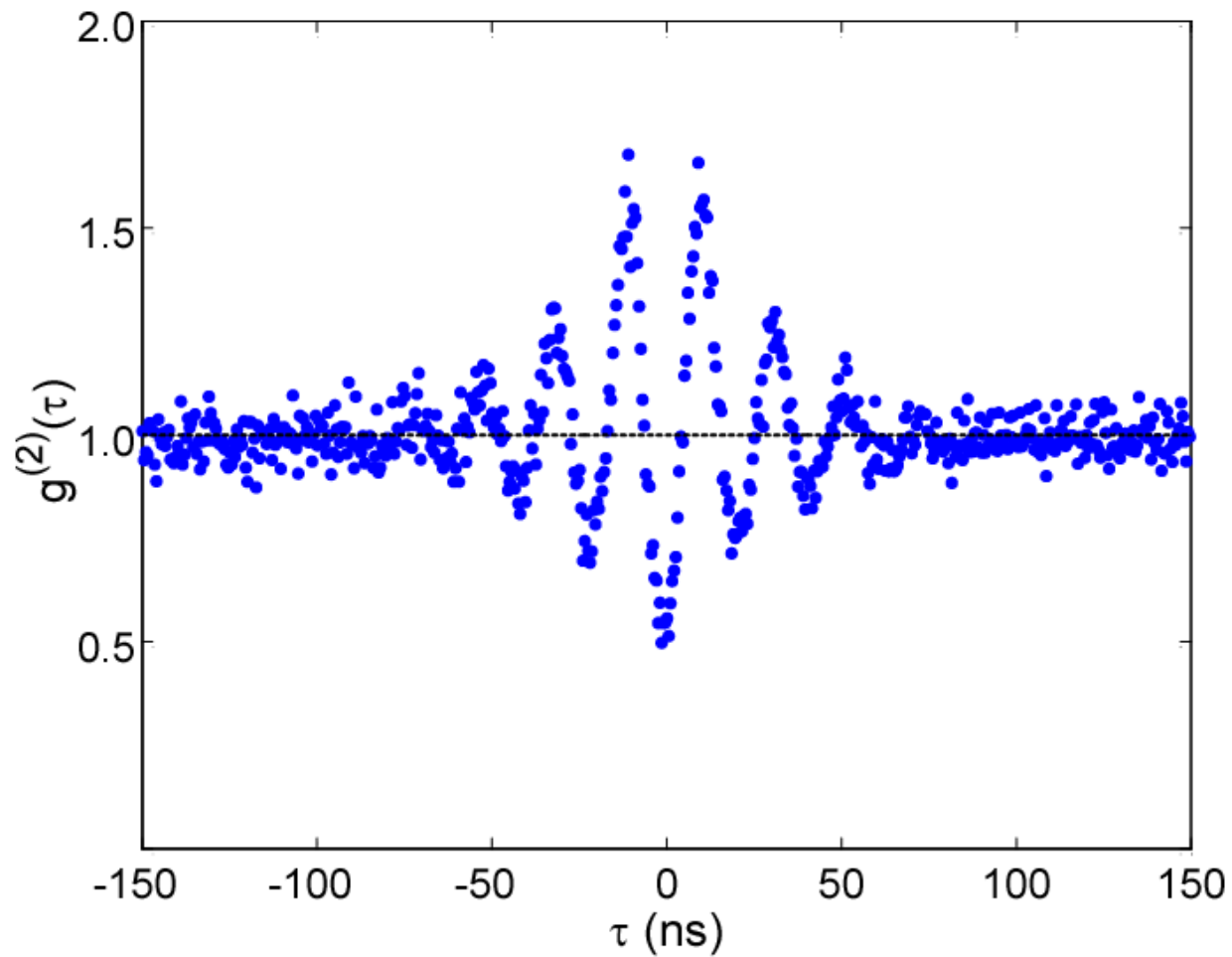
mean = 424



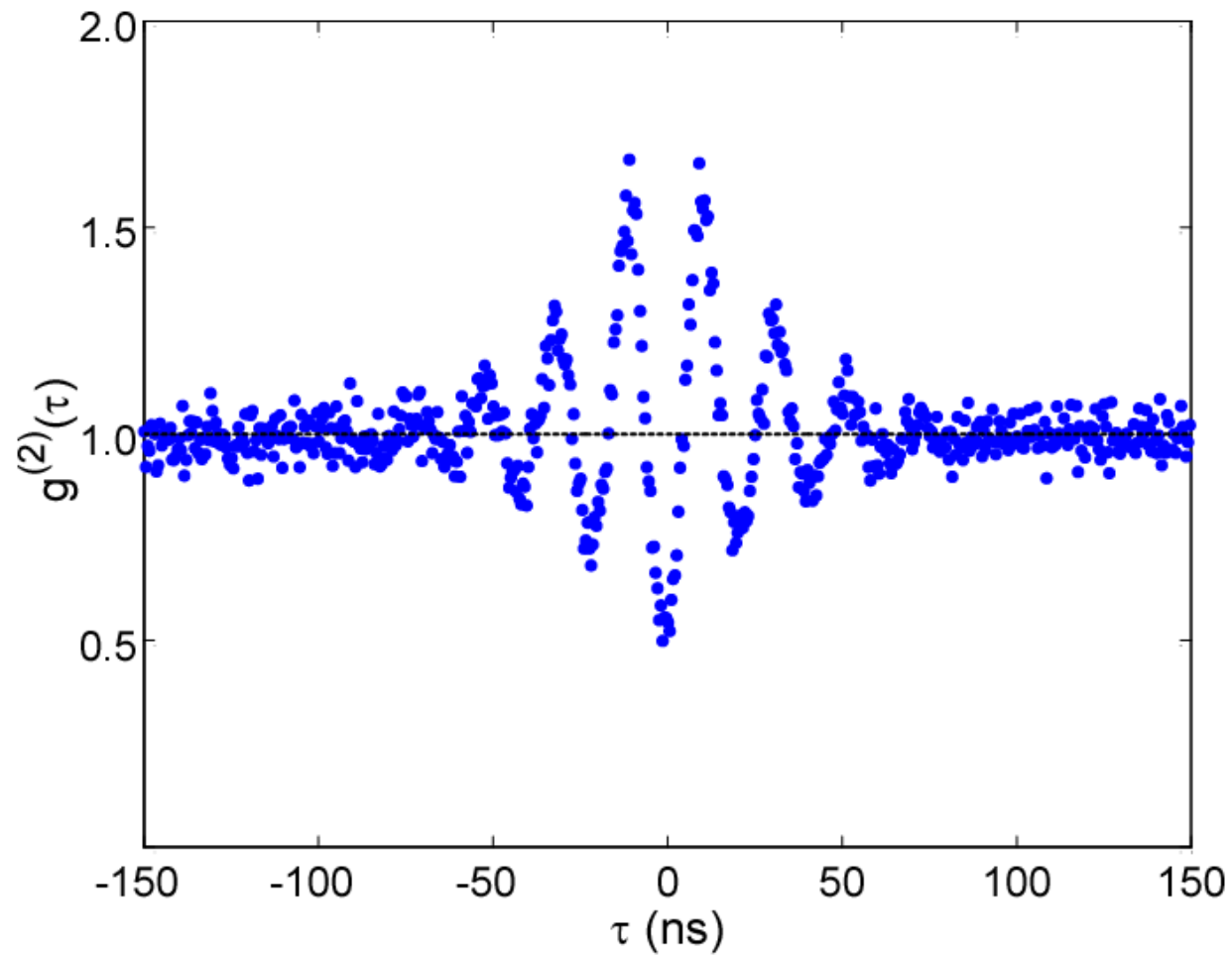
mean = 548



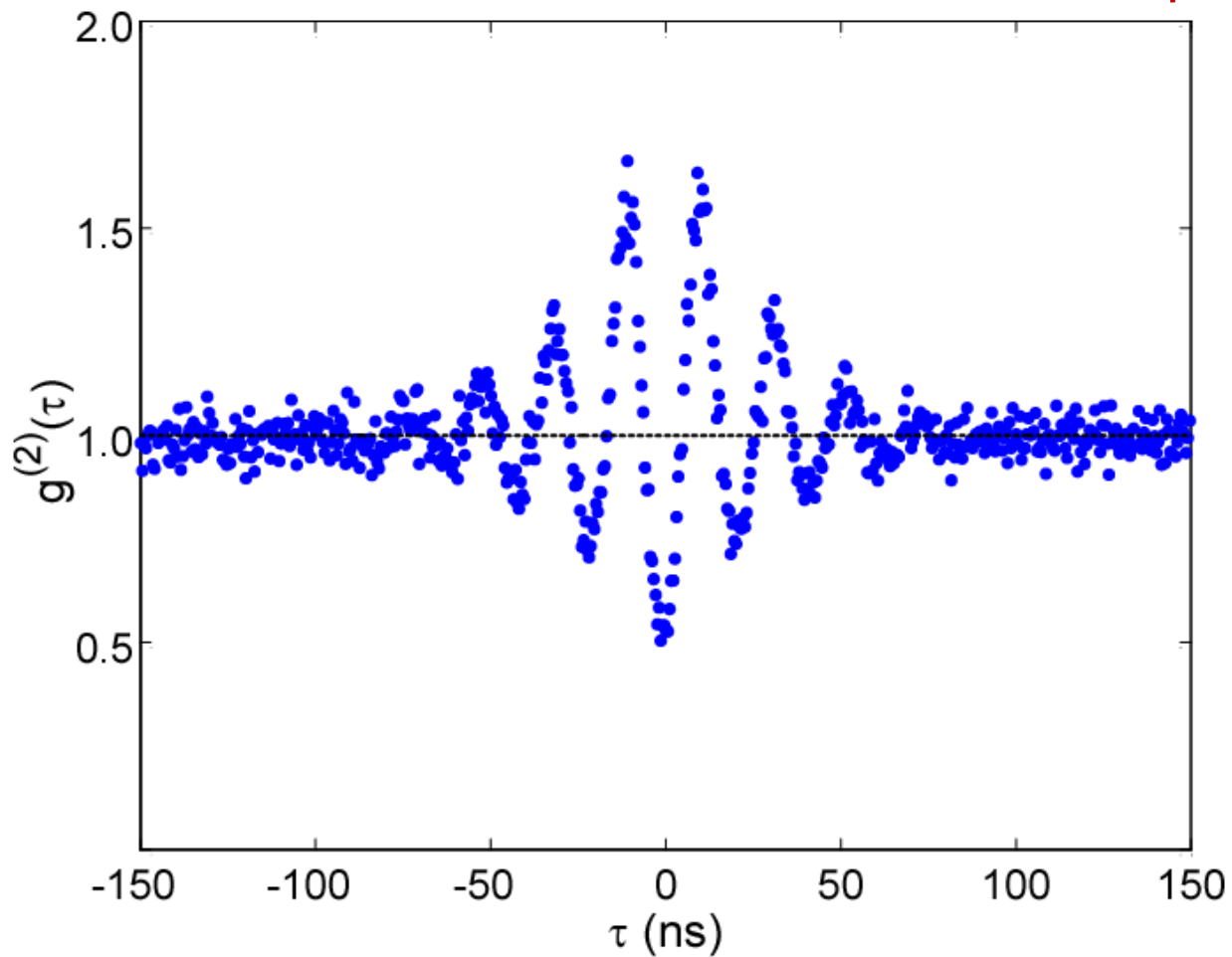
mean = 670

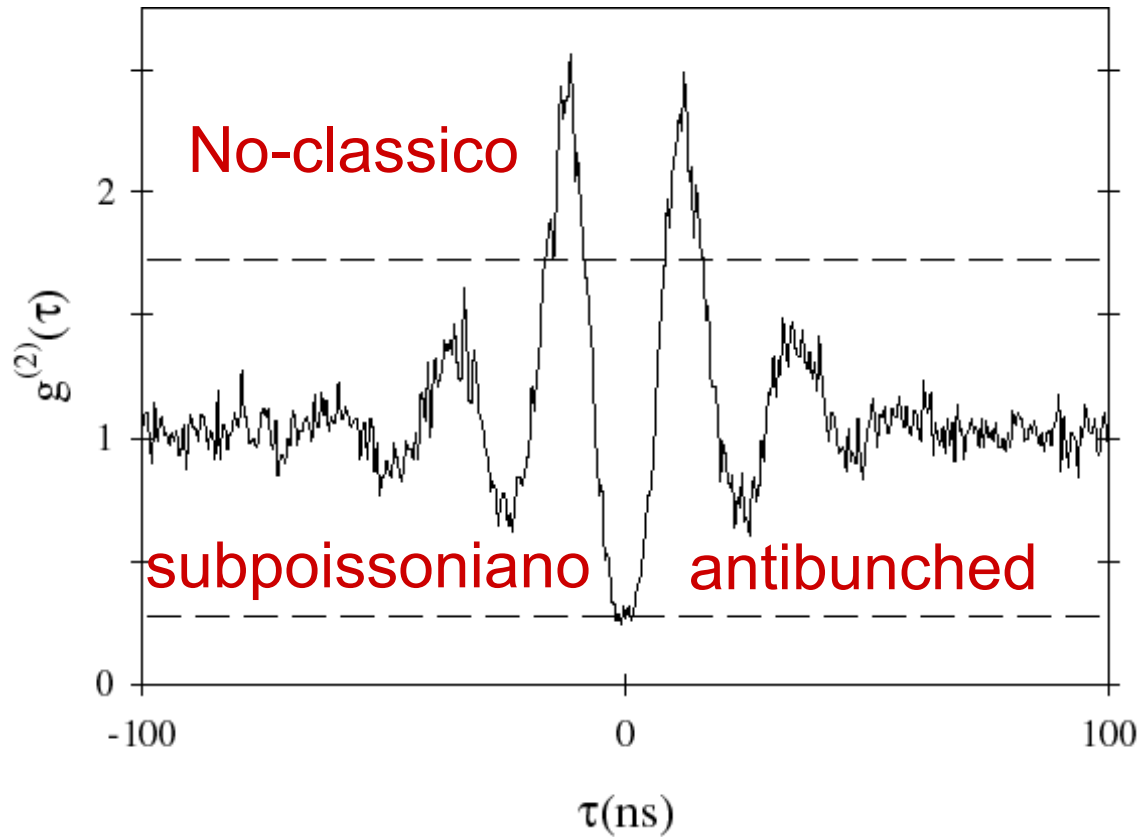


mean = 792



7 663 536 starts mean = 913 1 838 544 stops





Clásicamente $g^{(2)}(0) > g^{(2)}(\tau)$, $g^{(2)}(0) > g^{(2)}(0+\varepsilon)$ pendiente negativa
 antibunching, $g^{(2)}(0) < 1$ subpoissoniano, y $|g^{(2)}(0) - 1| > |g^{(2)}(\tau) - 1|$

Evolución condicional del estado un átomo

$$|\Psi_{ss}\rangle = |0, g\rangle + \lambda|1, g\rangle - \frac{2g}{\gamma}\lambda|0, e\rangle + \frac{\lambda^2 pq}{\sqrt{2}}|2, g\rangle - \frac{2g\lambda^2 q}{\gamma}|1, e\rangle$$

$$\lambda = \langle \hat{a} \rangle, \quad p = p(g, \kappa, \gamma) \text{ and } q = q(g, \kappa, \gamma) \quad \lambda = \frac{\varepsilon}{\kappa} \left(\frac{1}{1+2C} \right)$$

$$p = 1 - 2C'_1, \quad q = (1 + 2C) / (1 + 2C - 2C'_1) \quad \text{with } C'_1 = C_1(1 + \gamma/2\kappa)^{-1}$$

$$\hat{a}|\Psi_{ss}\rangle \Rightarrow |\Psi_{\text{conditioned}}\rangle = |0, g\rangle + \lambda pq|1, g\rangle - \frac{2g\lambda q}{\gamma}|0, e\rangle$$

$$|\langle 0, g | \hat{a}^2 | \psi \rangle| = |\lambda^2 pq|^2$$

↖
↖
Campo polarización atómica

$$g^{(2)}(0) = |pq|^2 = 1 - \frac{4C_1^2}{(1 + \gamma/2\kappa)(1 - 2C_1) - 2C_1}$$

Para N átomos que no interactúan

$$|\chi(t)\rangle = |00\rangle + A_1(t)|10\rangle + A_2(t)|01\rangle \\ + A_3(t)|20\rangle + A_4(t)|11\rangle + A_5(t)|02\rangle$$

$$\dot{A}_1 = -\kappa A_1 + \sqrt{N}gA_2 + \mathcal{E} \quad \text{Campo con forzamiento } \mathcal{E}$$

$$\dot{A}_2 = -(\gamma/2)A_2 - \sqrt{N}gA_1 \quad \text{Polarización}$$

$$\dot{A}_3 = -2\kappa A_3 + \sqrt{2}\sqrt{N}gA_4 + \sqrt{2}\mathcal{E}A_1,$$

$$\dot{A}_4 = -(\kappa + \gamma/2)A_4 - \sqrt{2}\sqrt{N}gA_3 \\ + \sqrt{2}\sqrt{N-1}gA_5 + \mathcal{E}A_2,$$

$$\dot{A}_5 = -\gamma A_5 - \sqrt{2}\sqrt{N-1}gA_4.$$

Ecuaciones de
movimiento de
los coeficientes

A. Low-intensity theory for $g^{(2)}(\tau)$

$$g^{(2)}(\tau) = |1 + \mathcal{A}\mathcal{F}(\tau)|^2, \quad (17)$$

where \mathcal{F} is a decaying oscillation,

$$\mathcal{F} = e^{-\beta\tau} [\cos(\Omega_0\tau) + (\beta/\Omega_0)\sin(\Omega_0\tau)], \quad (18)$$

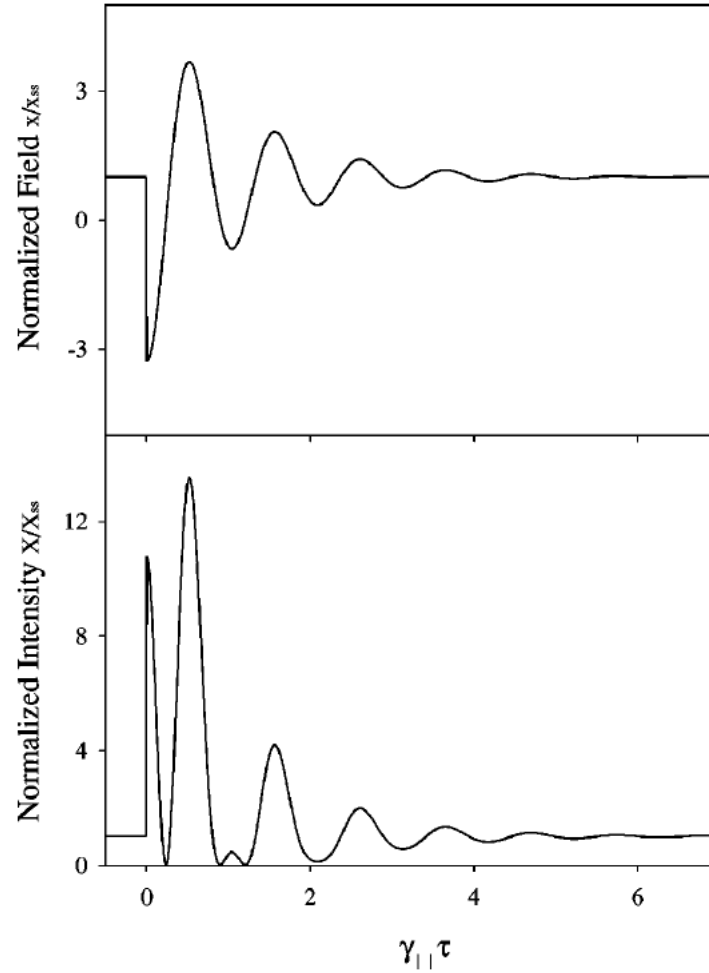
with $\beta \equiv (\kappa + \gamma_{\perp})/2$ representing the average decay rate and Ω_0 the vacuum Rabi frequency in the low intensity limit:

$$\Omega_0 = \sqrt{g^2 N - \frac{(\kappa - \gamma_{\perp})^2}{4}}.$$

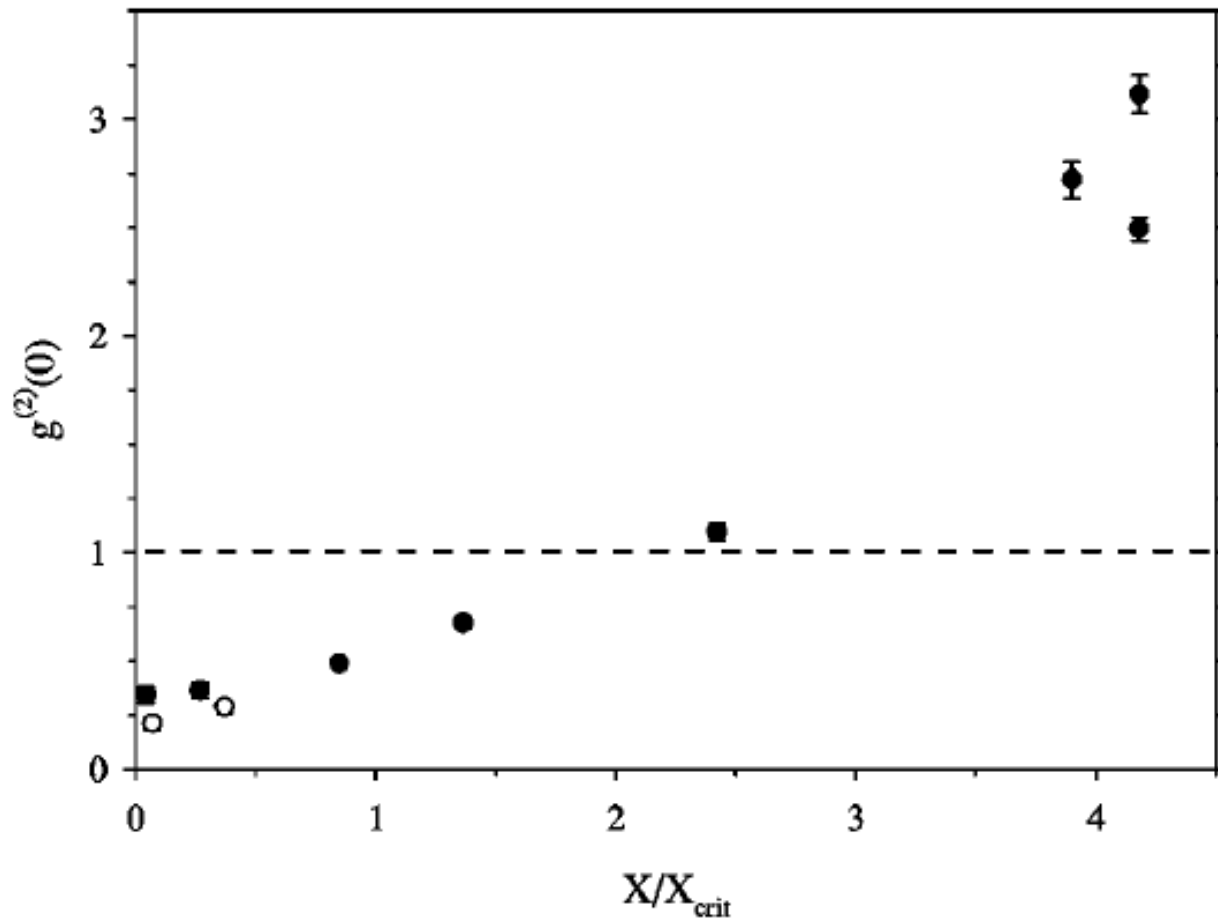
The amplitude of the decaying oscillations is given by

$$\mathcal{A} = - \frac{4C_1^2 N}{(1 + \gamma_{\perp}/\kappa)(1 + 2C_1 N) - 2C_1}.$$

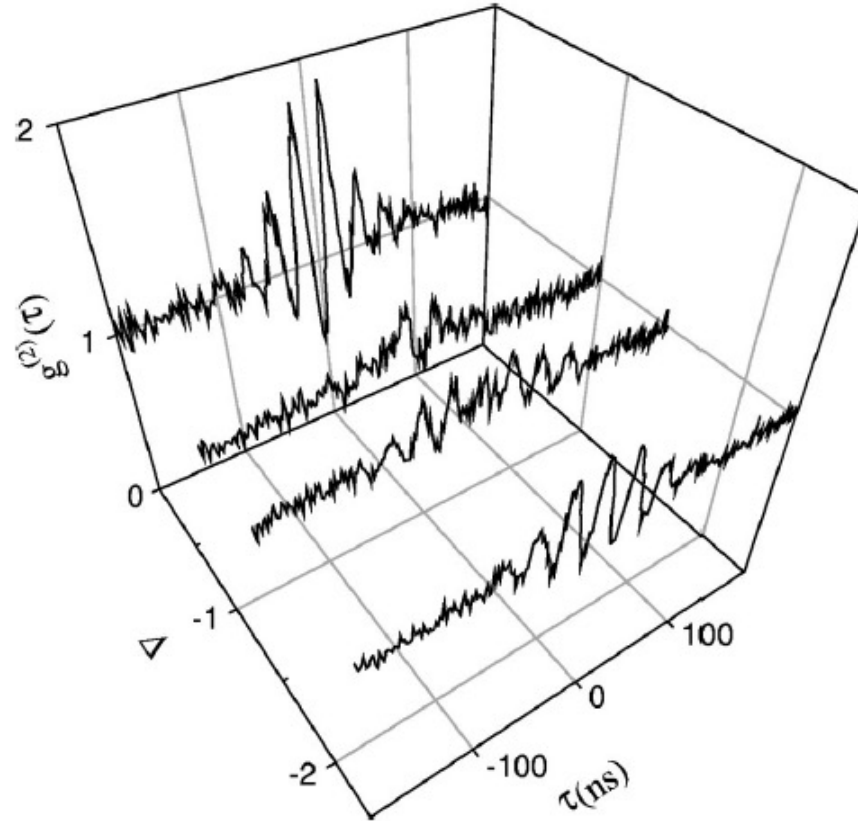
¿Qué sucede después de un clic?



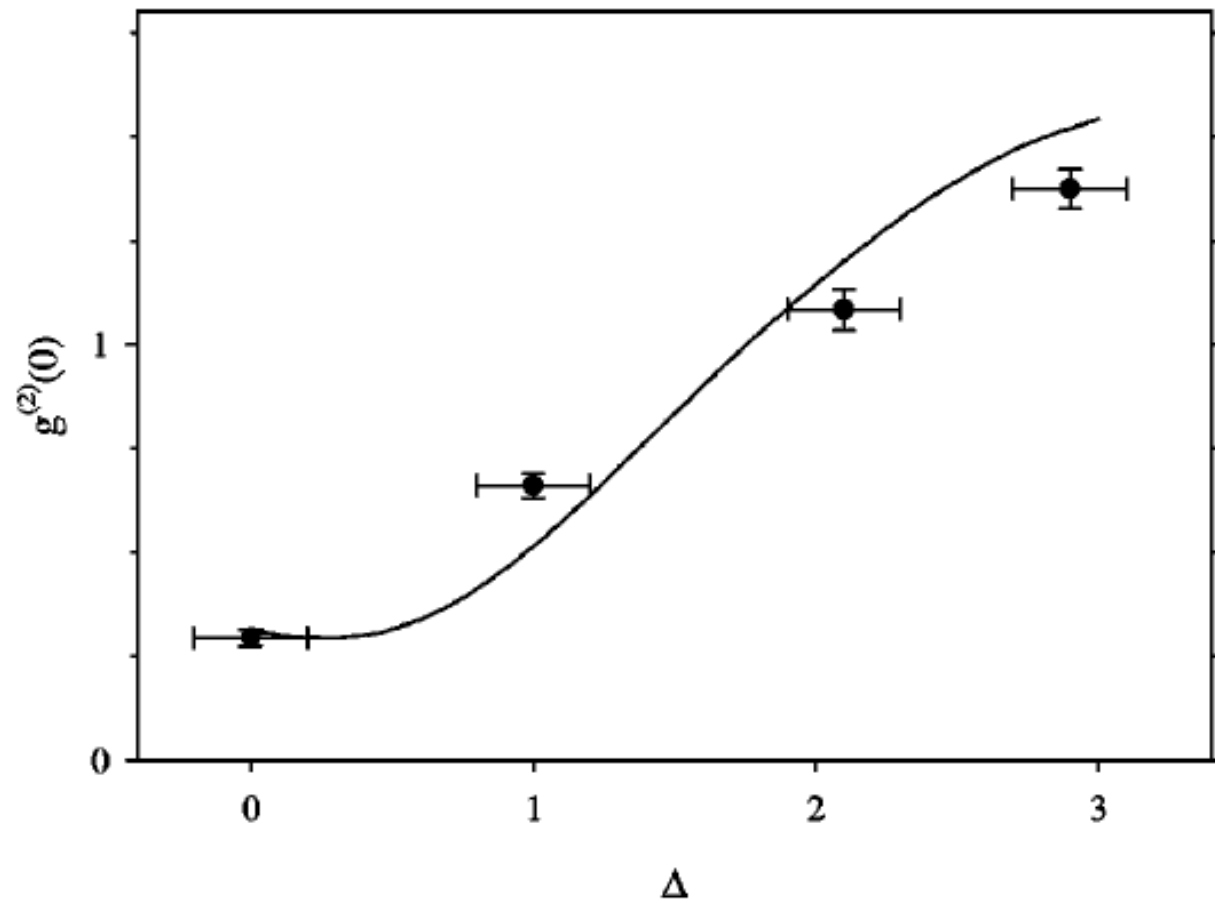
En función de la intensidad de forzamiento



En función del desencaje



Dessencaje



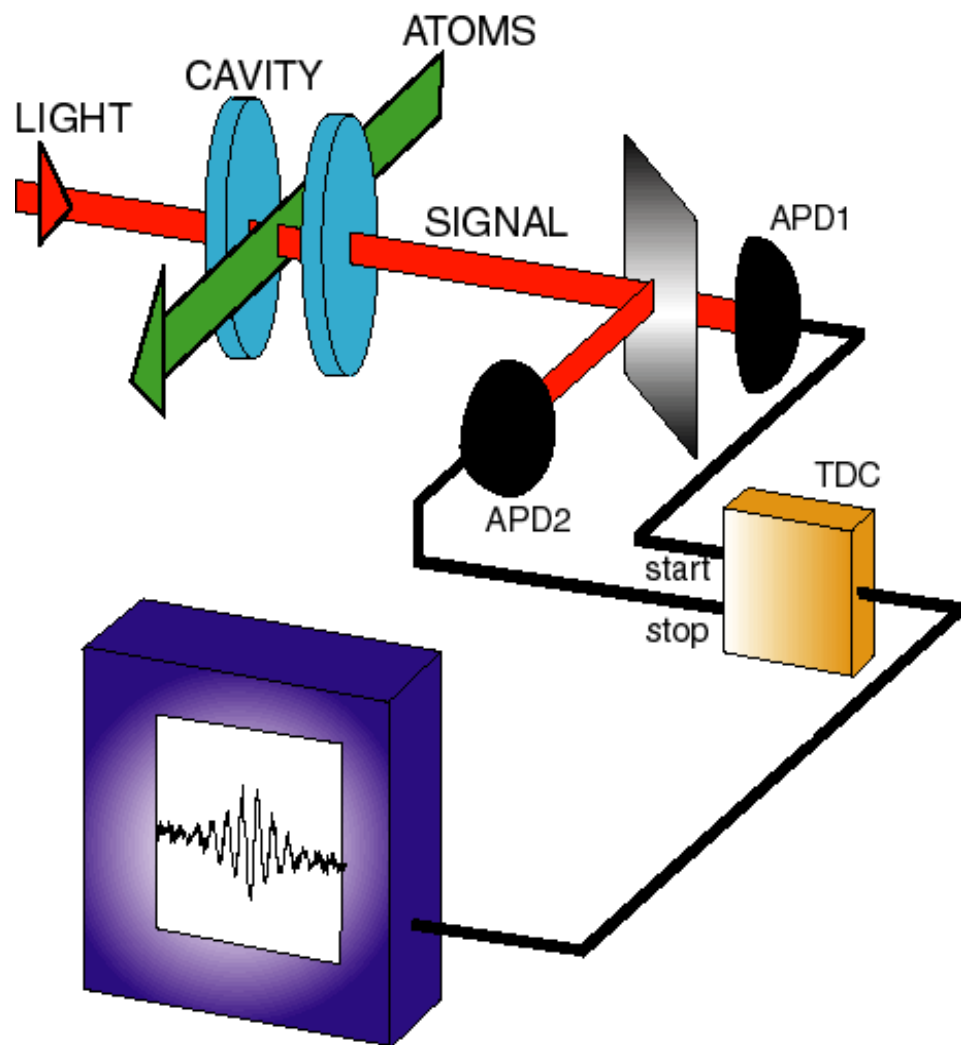
Bibliografía:

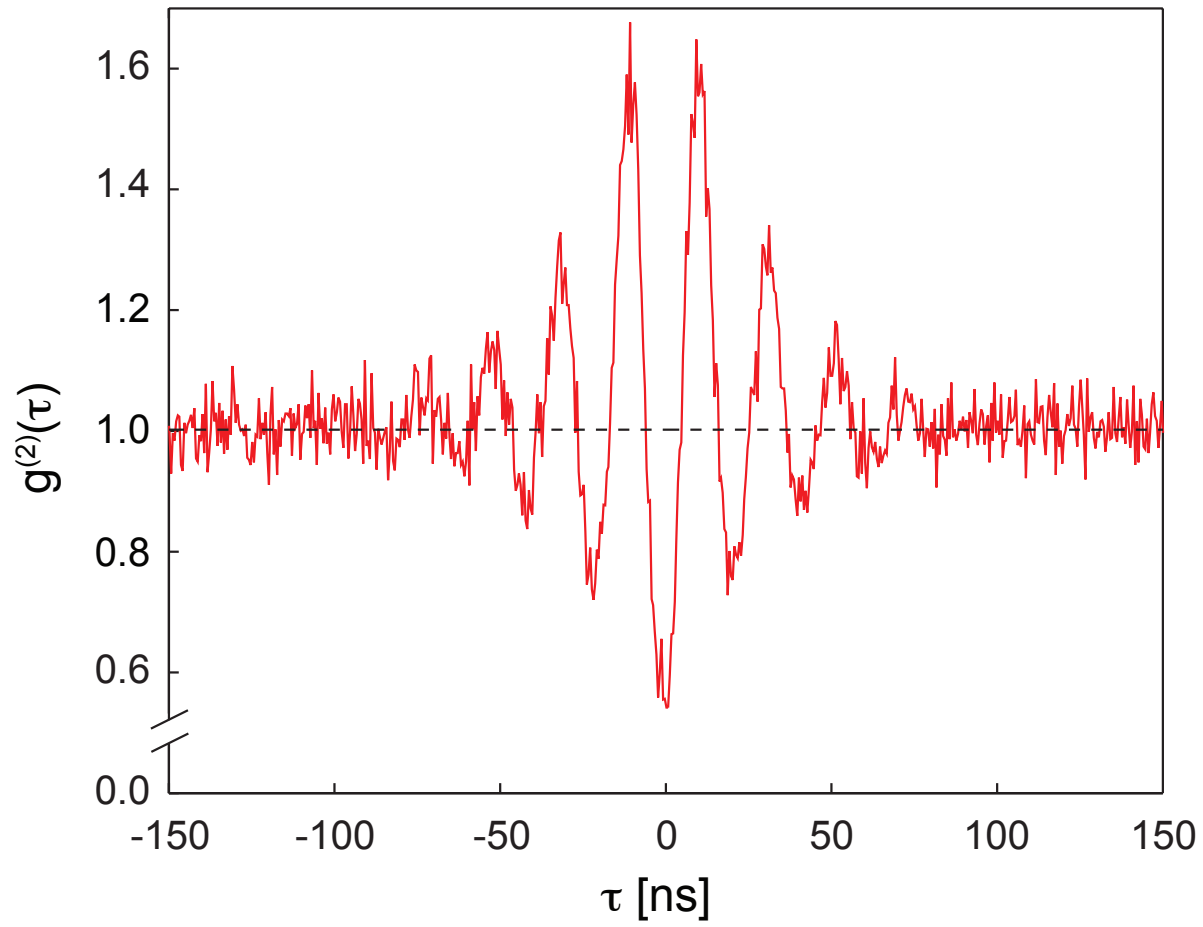
H. J. Carmichael, R. J. Brecha, P. R. Rice, “Quantum interference and collapse of the wavefunction in cavity QED,” *Optics Communications*, **82**, 73 (1991).

S. L. Mielke, G. T. Foster, and L. A. Orozco, “Non-classical intensity correlations in cavity QED,” *Phys. Rev. Lett.* **80**, 3948 (1998).

G. T. Foster, S. L. Mielke, and L. A. Orozco, “Intensity correlations in cavity QED,” *Phys. Rev. A*, **61**, 053821 (2000).

Retroalimentación en la cavity QED





Evolución condicional del estado de N átomos

$$|\psi_{ss}\rangle = |0, G\rangle + \lambda \left(|1, G\rangle - \frac{2g\sqrt{N}}{\gamma} |0, E\rangle \right)$$

Estado estable

$$+ \lambda^2 \left(\zeta_0 \frac{1}{\sqrt{2}} |2, G\rangle - \theta_0 \frac{2g\sqrt{N}}{\gamma} |1, E\rangle \right) + \dots$$

Detección de un fotón

$$\hat{a} |\psi_{ss}\rangle / \sqrt{\langle \hat{a}^\dagger \hat{a} \rangle_{ss}},$$

Estado condicional

$$|\psi_c(\tau)\rangle = |0, G\rangle + \lambda \left(\zeta(\tau) |1, G\rangle - \theta(\tau) \frac{2g\sqrt{N}}{\gamma} |0, E\rangle \right)$$

$$+ O(\lambda^2).$$

$$\lambda = \langle \hat{a} \rangle = \frac{\varepsilon}{\kappa} \left(\frac{1}{1+2C} \right)$$

Algoritmo

Estado condicional

$$|\psi_c(\tau)\rangle = |0, G\rangle + \lambda \left(\zeta(\tau) |1, G\rangle - \theta(\tau) \frac{2g\sqrt{N}}{\gamma} |0, E\rangle \right) + O(\lambda^2).$$

Si elegimos un tiempo $\tau=T$ tal que $\zeta(T) = \theta(T)$ a orden λ obtenemos un estado estable con una nueva λ'

$$|\psi_c(T)\rangle \simeq |0, G\rangle + \lambda' \left(|1, G\rangle - \frac{2g\sqrt{N}}{\gamma} |0, E\rangle \right)$$

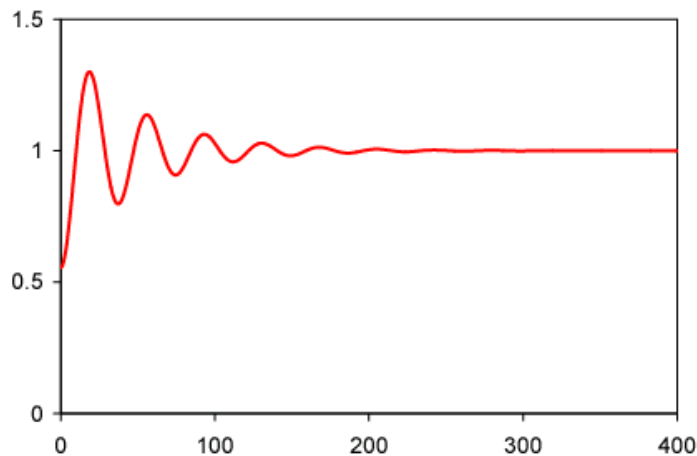
Entonces, en presencia de retroalimentación, la función de correlación no es simétrica, pero aún así se calcula correctamente mediante:

$$g^{(2)}(\tau) \simeq \frac{|\langle 1, G | \psi_{c+fb}(\tau) \rangle|^2}{|\langle 1, G | \psi_{ss} \rangle|^2} = [\zeta_{fb}(\tau)]^2$$

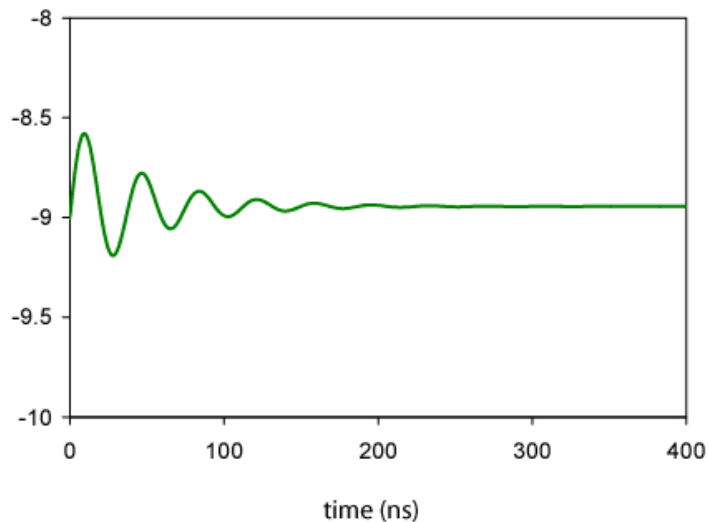
El tiempo T está cerca del momento en que la fluctuación del campo cruza la media. Esta forma de estabilizar el estado condicional es posible porque es un estado cuántico puro con dos parámetros reales. (θ y ζ) y dos parámetros de control: el cambio en la unidad $\lambda' - \lambda$ y el momento del cambio T .

$$\lambda = \langle \hat{a} \rangle = \frac{\varepsilon}{\kappa} \left(\frac{1}{1+2C} \right)$$

Campo $\sim \zeta(\tau)$



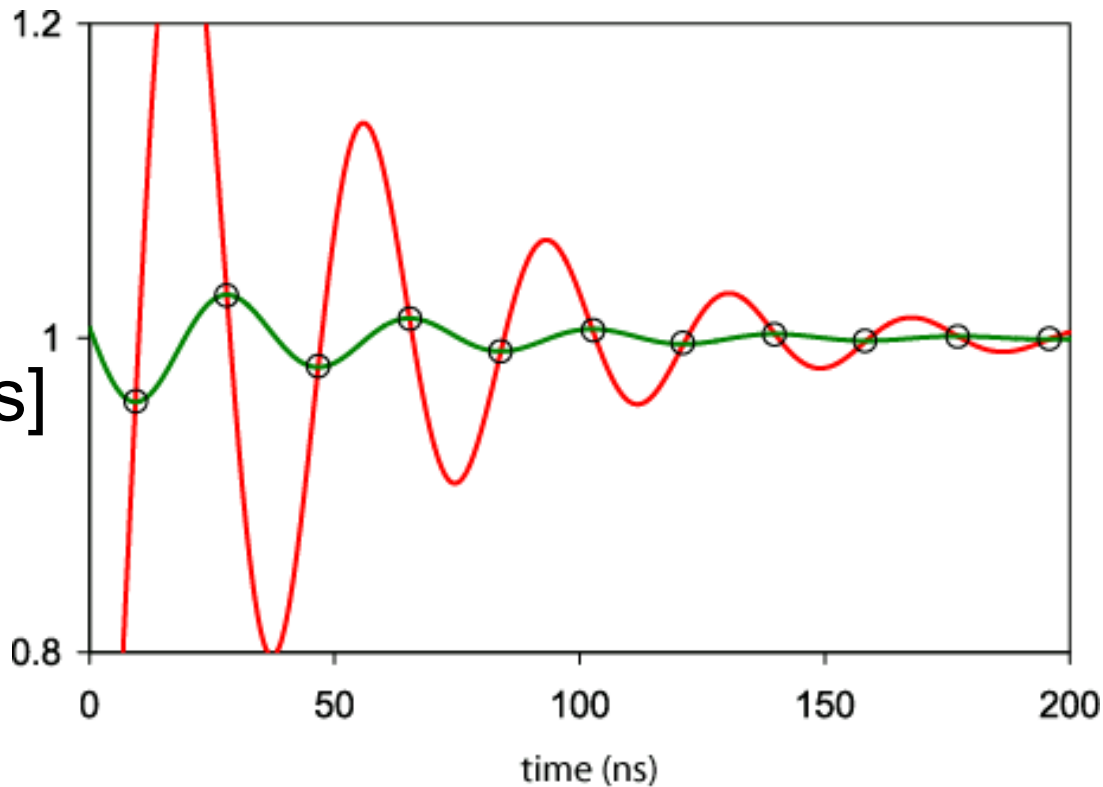
Polarización
atómica $\sim \theta(\tau)$



$$f_2(T) = -\frac{2g\sqrt{N}}{\gamma} f_1(T)$$

$$\lambda(t > T) = f_1(T)\lambda(t < T)$$

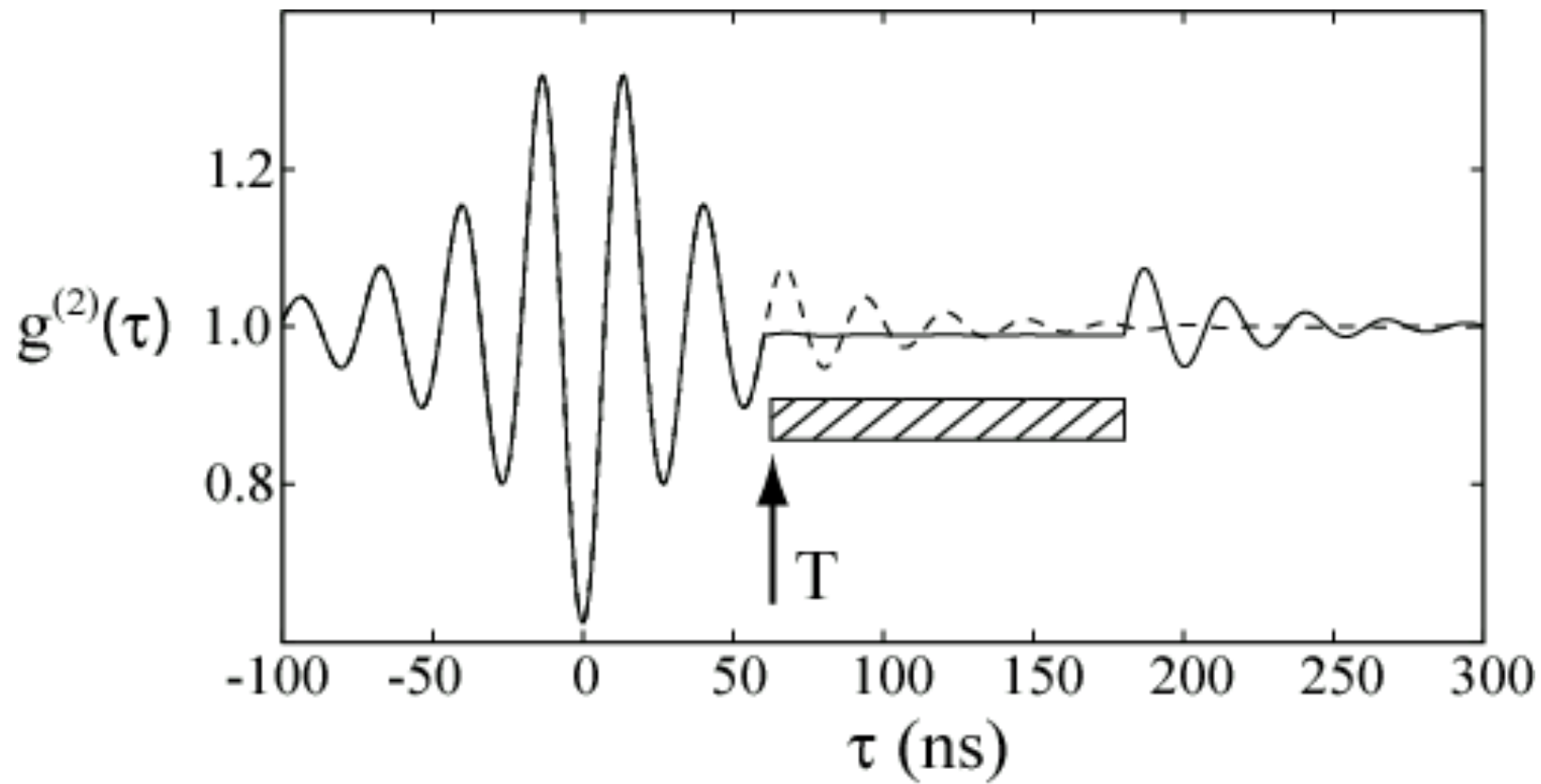
$\zeta(\tau)$ [Campo rojo]
 $\theta(\tau)$ [átomos verdes]



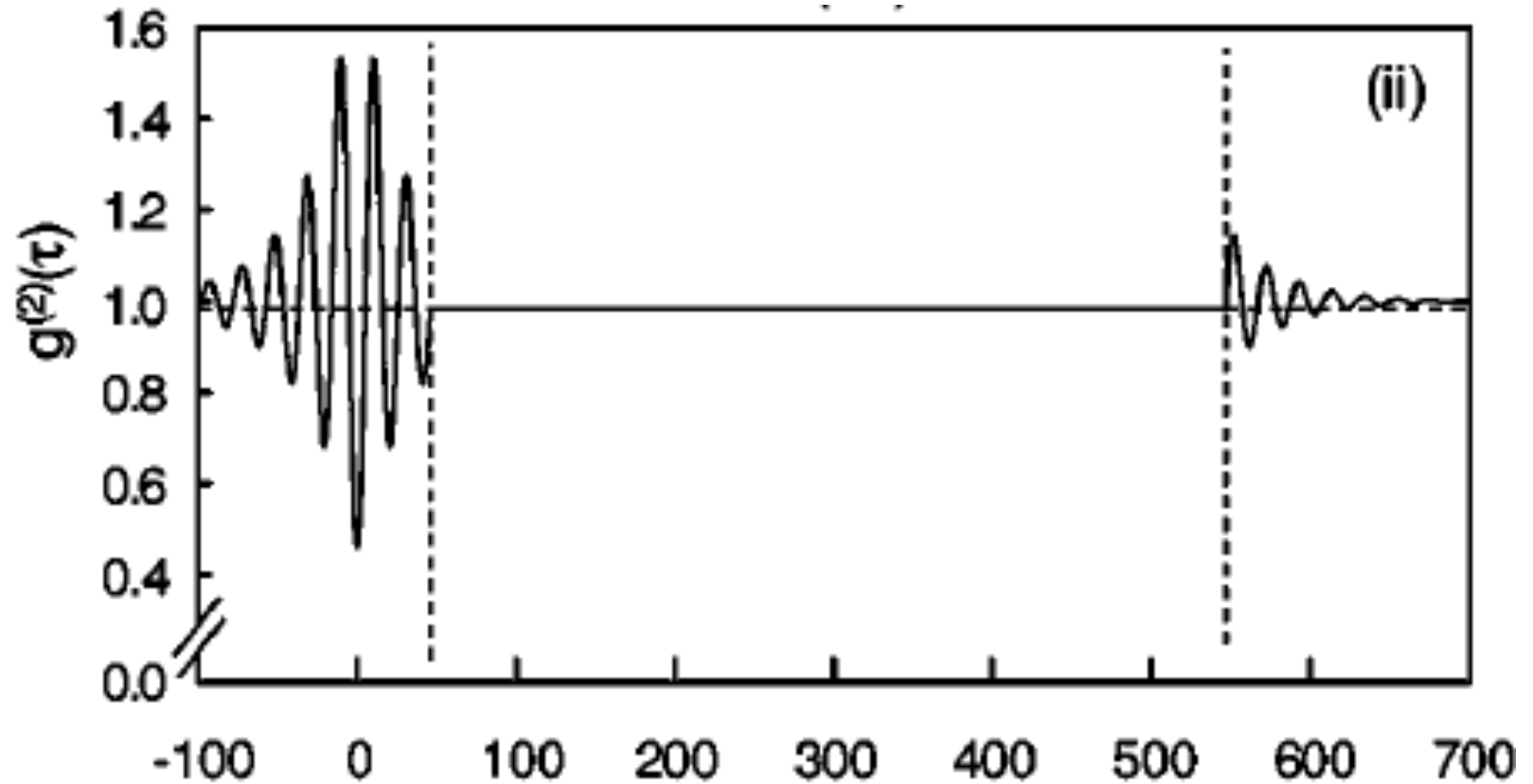
Cruce en:

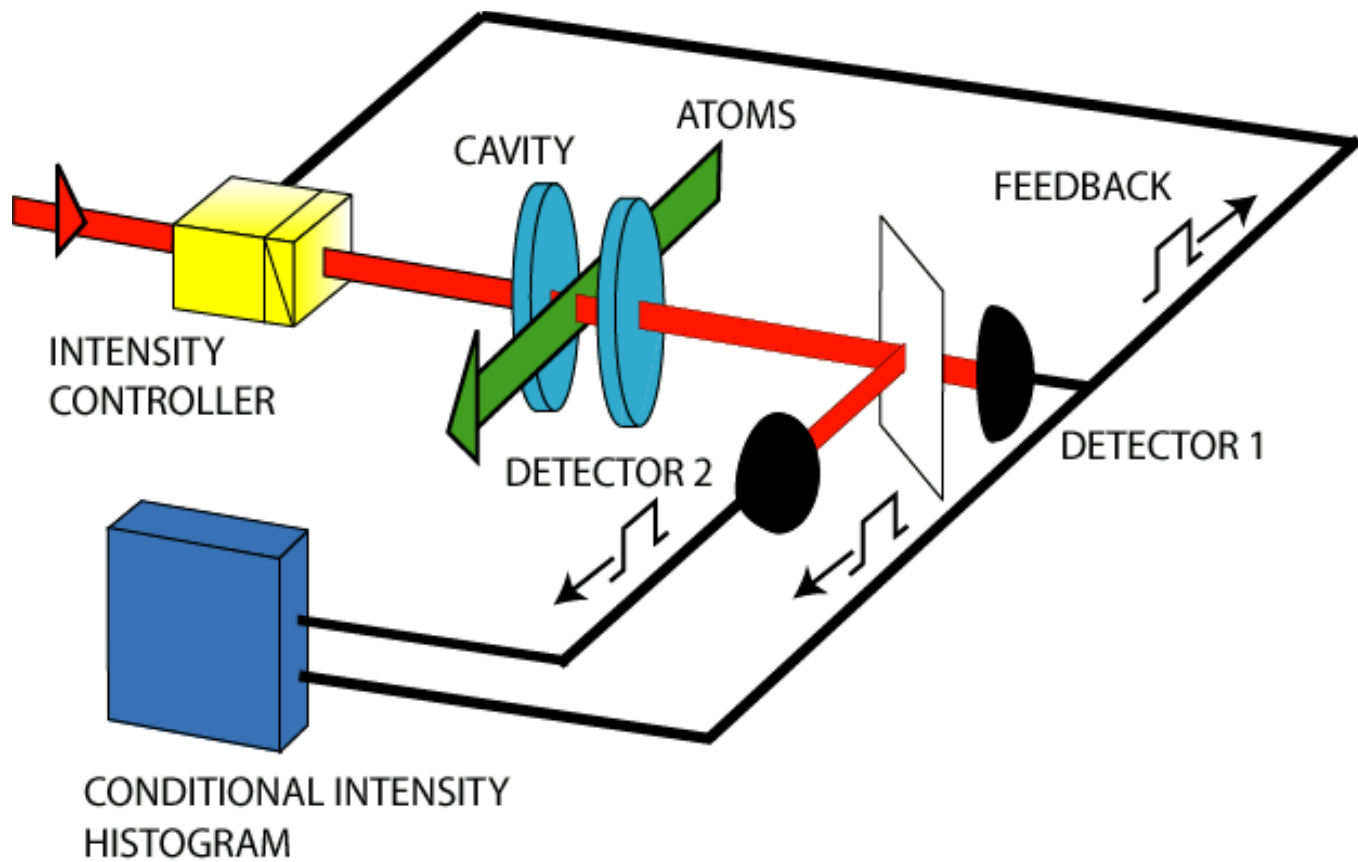
$$\Omega T = n\pi - \tan^{-1} \left(\frac{1 + 2C}{2C \left(\frac{2\kappa + \gamma}{4\Omega} \right) + \frac{4\kappa^2 - \gamma^2 - 16\Omega^2}{16\Omega\kappa}} \right)$$

Predicción teórica.



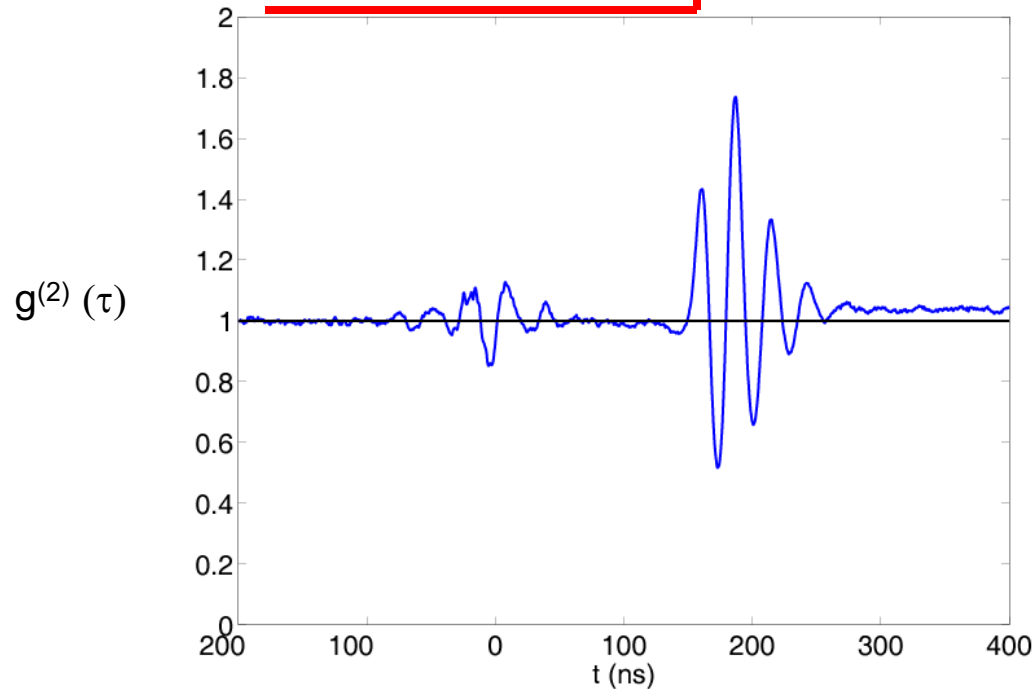
Captura y liberación





Intensidad condicional y paso:

Primera detección. ↓ ↓ Comienzo del pulso



Tres condiciones:

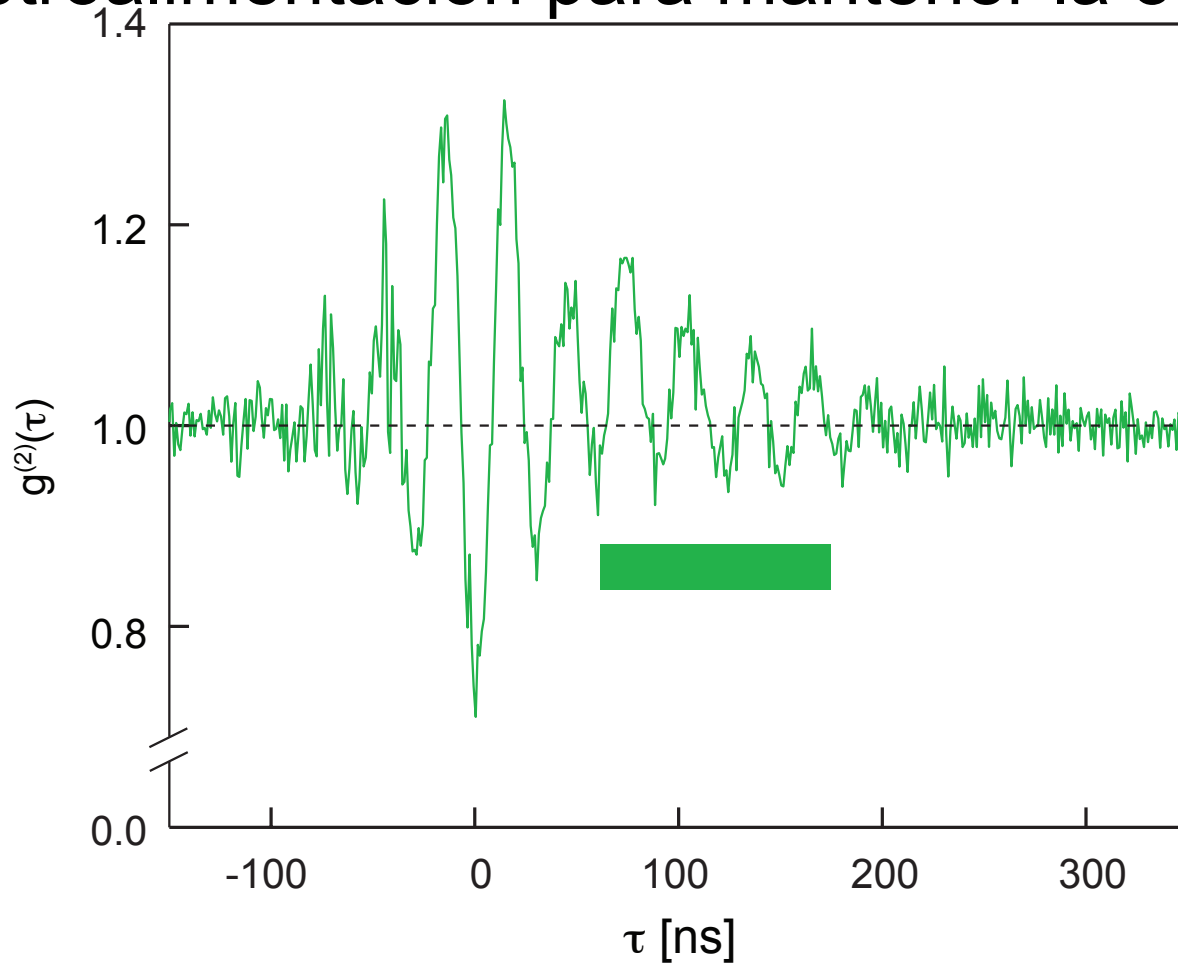
Amplitud

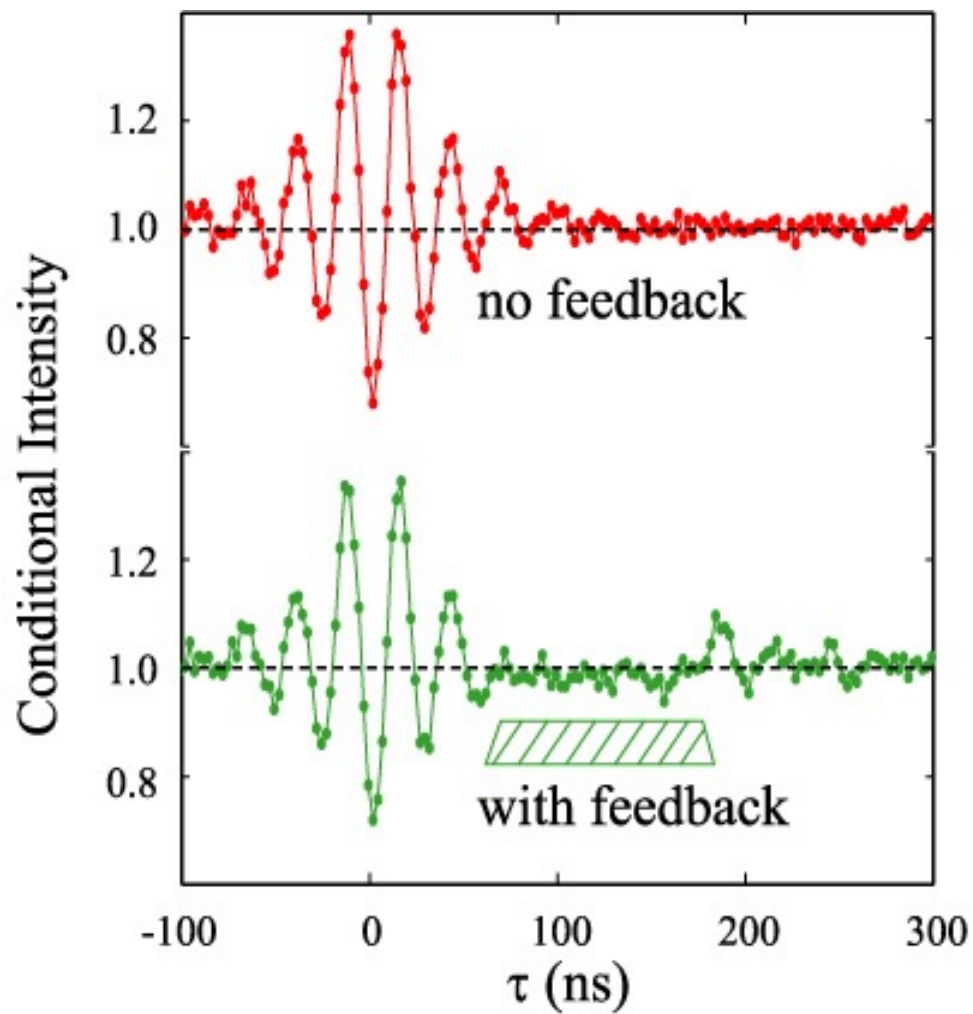
Paridad

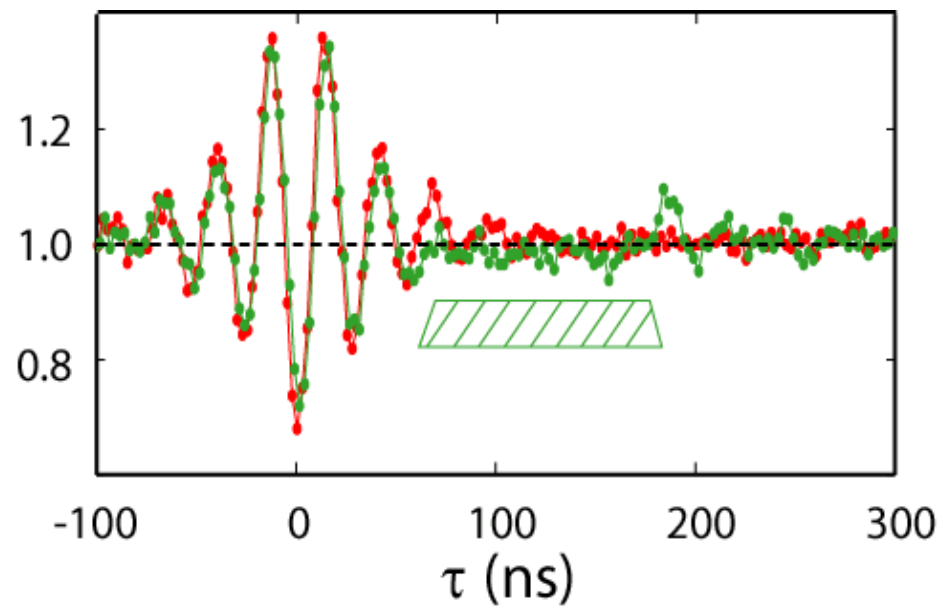
Tiempo de empuje

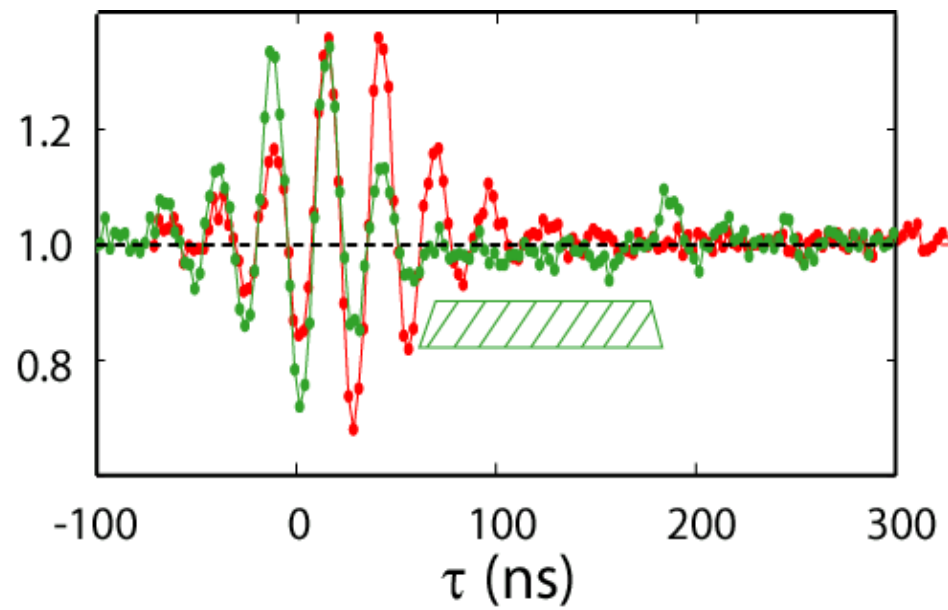
Solo tenemos un bit de información, un clic.
Pero tenemos una muy buena comprensión de
la dinámica.

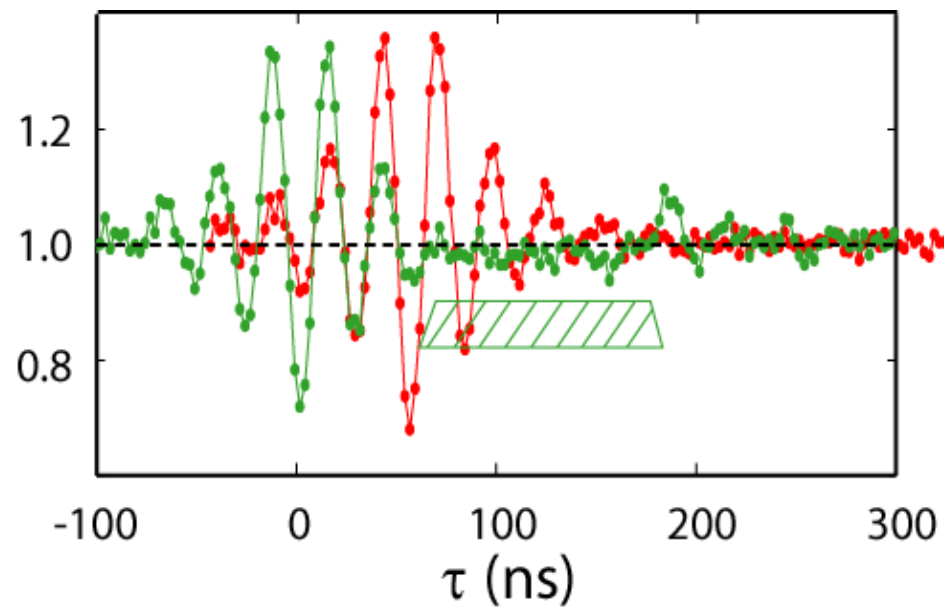
Con retroalimentación para mantener la oscilación

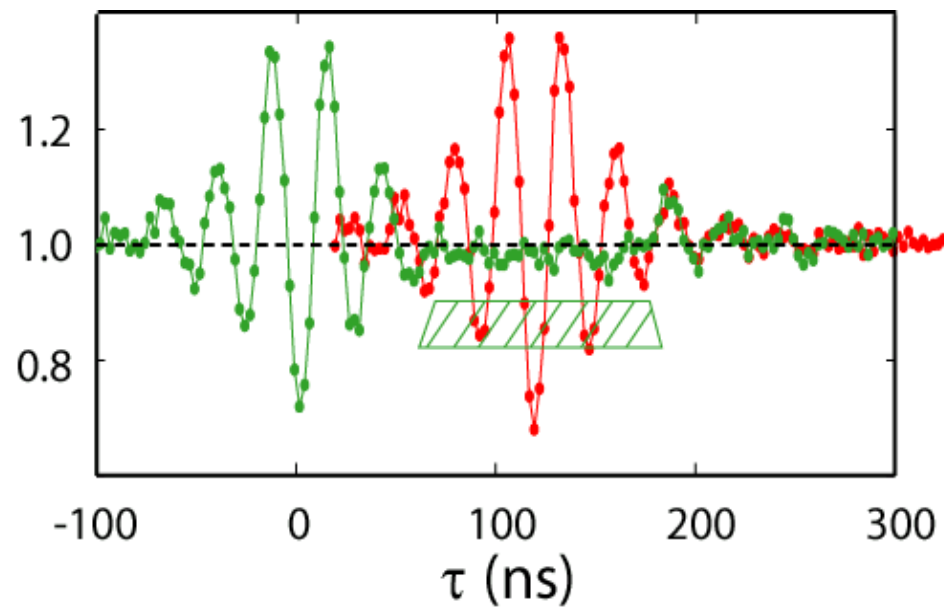




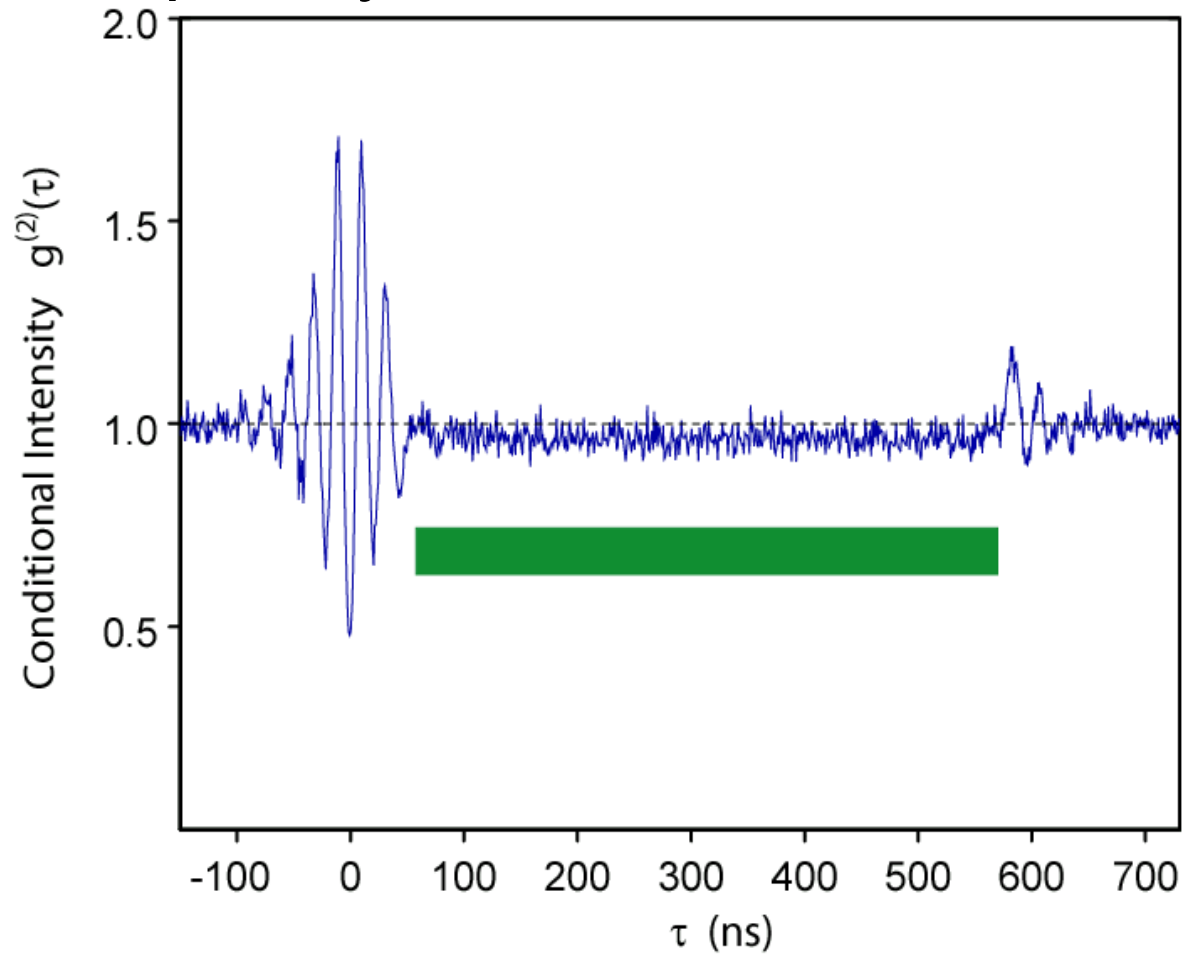








Captura y liberación de oscilación



¿Cuánto tiempo podemos mantener el sistema?

Tanto como queramos

¿Dónde está la información?

Hay un nuevo estado estacionario.

¿Qué tiene esto de cuántico?

La detección del primer fotón que desencadena la evolución condicional.

Bibliografía

W. P. Smith, J. E. Reiner, L. A. Orozco, S. Kuhr, H. M. Wiseman, "Capture and release of a conditional state of a cavity QED system by quantum feedback," Phys. Rev. Lett. **89**, 133601, (2002).

J. E. Reiner, W. P. Smith, L. A. Orozco, H. M. Wiseman, and Jay Gambetta, "Quantum feedback in a weakly driven cavity QED system," Phys. Rev. A. **70**, 023819, (2004).

Gracias

Correlations of the intensity at $\tau=0$

$$\begin{aligned}g^{(2)}(0) &= \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2} \\ &= \frac{\langle (I_0 + \delta(t))^2 \rangle}{\langle I_0 + \delta(t) \rangle^2} \\ &= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}\end{aligned}$$

It is proportional to the variance

Intensity correlations (bounds)

$$g^{(2)}(0) = 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

$$g^{(2)}(0) - 1 \geq 0$$

Cauchy-Schwarz

$$2I(t)I(t + \tau) \leq I^2(t) + I^2(t + \tau)$$

$$|g^{(2)}(\tau) - 1| \leq |g^{(2)}(0) - 1|$$

The correlation is maximal at equal times ($\tau=0$) and it can not increase.

Quantum Correlations (Glauber):

$$g^{(2)}(\tau) = \frac{\langle T : \hat{I}(t) \hat{I}(t + \tau) : \rangle}{\langle \hat{I}(t) \rangle^2}$$

The intensity operator I is proportional to the number of photons, but the operators have to be normal ($:$) and time (T) ordered. All the creation operators do the left and the annihilation operators to the right (just as a photodetector works). The operators act in temporal order.

R. Glauber, "*The Quantum Theory of Optical Coherence*," Phys. Rev. **130**, 2529 (1963).

At equal times (normal order) :

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}.$$

Commutator : $\hat{a}^\dagger \hat{a} = \hat{a} \hat{a}^\dagger - 1$

$$\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle = \langle \hat{a}^\dagger (\hat{a} \hat{a}^\dagger - 1) \hat{a} \rangle = \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle$$

$$\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle \quad \text{where} \quad \hat{n} = \hat{a}^\dagger \hat{a}$$

The correlation requires detecting two photons, so if we detect one, we have to take that into consideration in the accounting.

In terms of the variance of the photon number:

$$\sigma^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$$

$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}$$

The classical result says:

$$= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

The quantum correlation function can be zero, as the detection changes the number of photons in the field. This is related to the variance properties: is the variance larger or smaller than the mean (Poissonian, Super-Poissonian or Sub-Poissonian).

$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2},$$

At equal times the variance gives

$g^{(2)}(0)=1$ Poissonian

$g^{(2)}(0)>1$ Super-Poissonian

$g^{(2)}(0)<1$ Sub-Poissonian

The slope at time=0:

$g^{(2)}(0)>g^{(2)}(0^+)$ Bunched

$g^{(2)}(0)<g^{(2)}(0^+)$ Antibunched

Classically we can not have Sub-Poissonian nor Antibunched.

effective number of atoms

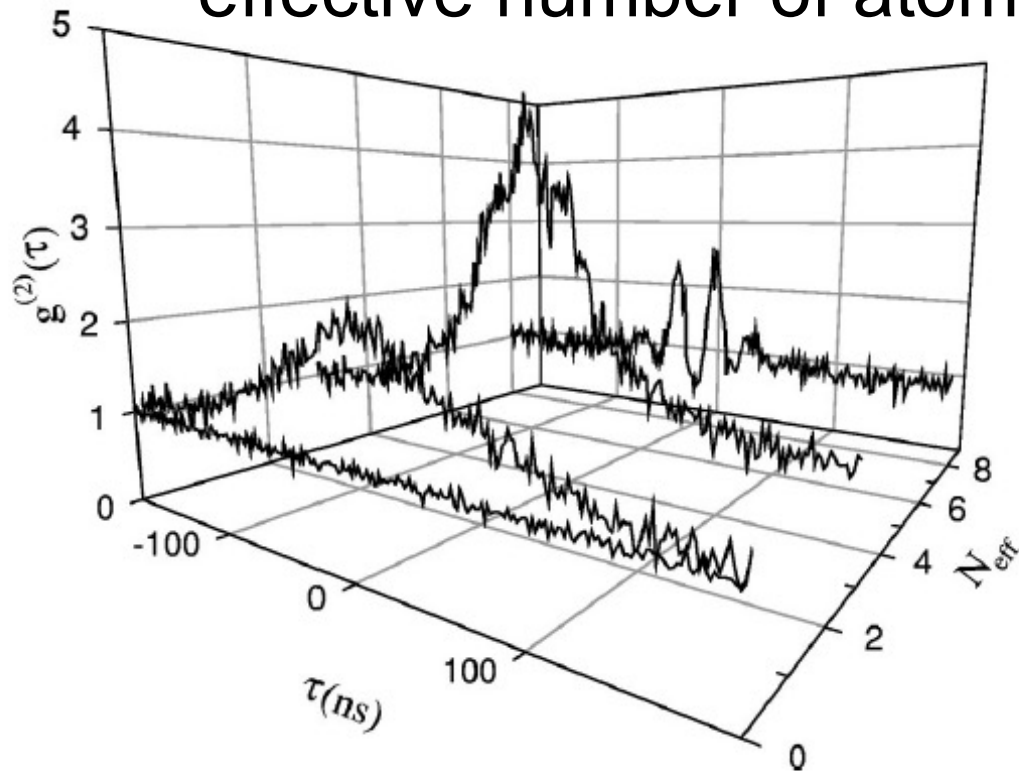


FIG. 11. Evolution of $g^{(2)}(\tau)$ as a function of the effective number of atoms for $N_{\text{eff}} = 0, 0.1, 3, \text{ and } 8$. Each correlation is taken on resonance. For the cases with atoms, the intracavity intensities

Large background
thermal atoms

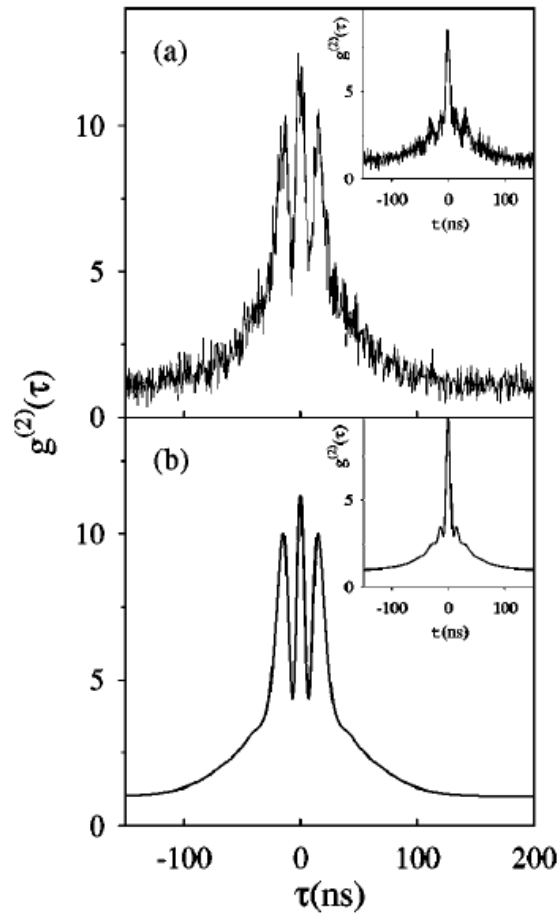
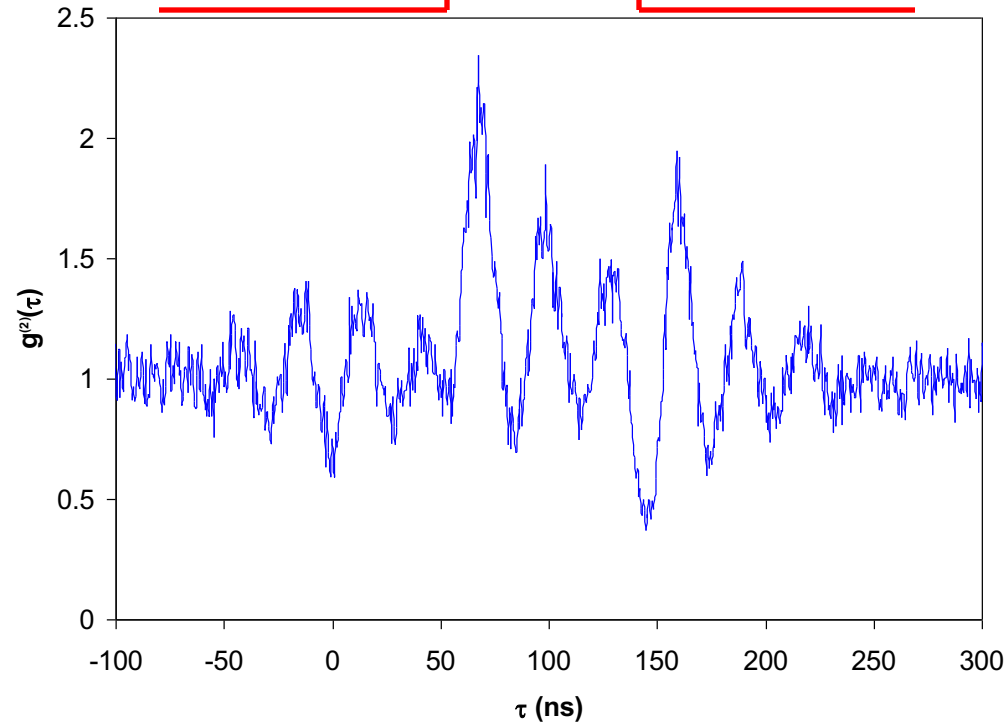


FIG. 10. $g^{(2)}(\tau)$ for cavity 3 with a small effective atom number $N \approx 3$, and $X = 0.06$. The main plot (a) shows data collected on resonance. The inset shows data with a detuning of $\Delta = 1.5$. Plot (b)

Inicio del pulso de retroalimentación

Primera detección.

Fin del pulso de retroalimentación



Implementación

